Introduction to Machine Learning

Lecture 15: Convolutional Neural Network (CNN)

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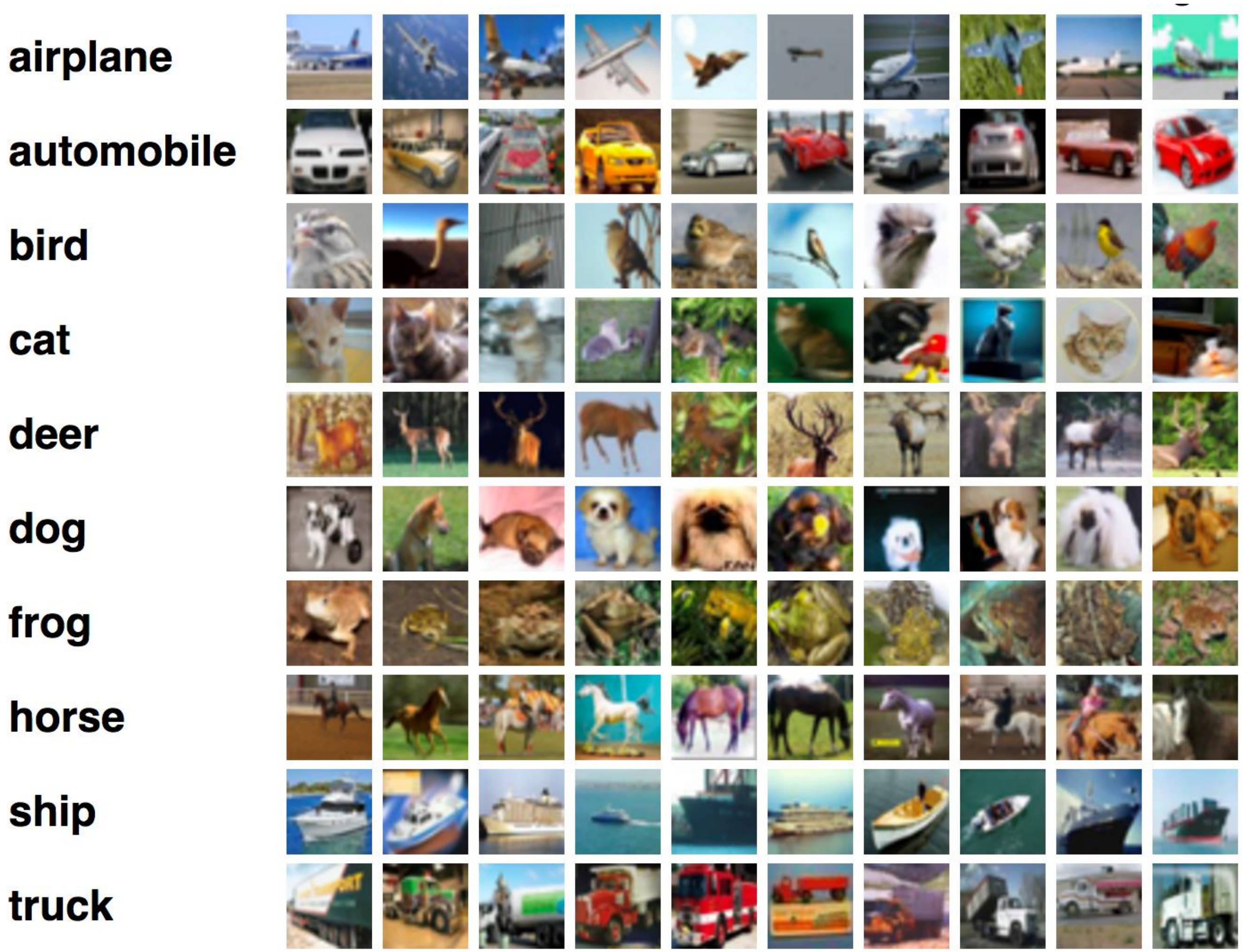
MiRA

Machine Intelligence Research and Applications Lab

Contents

Introduction

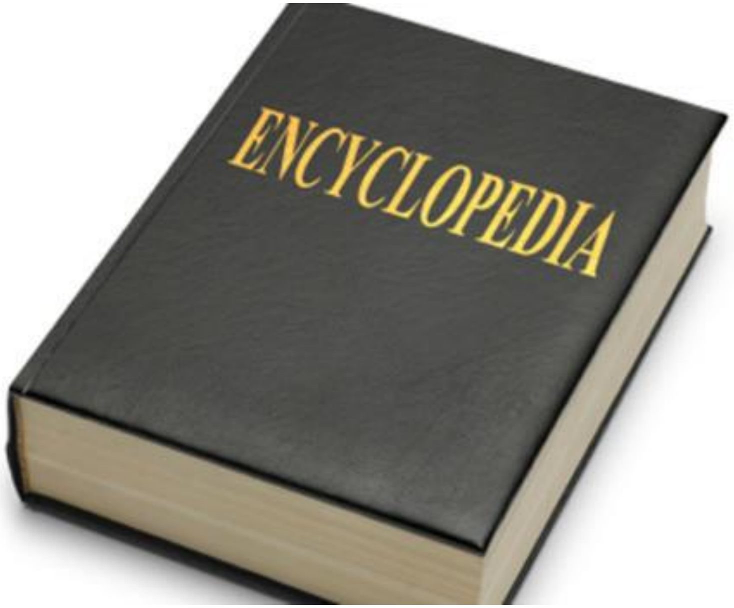
Image Classification

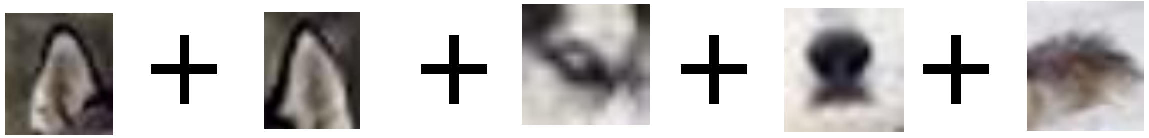


- Identifying objects from various scenes is a easy task for human
- However, it is difficult for human to describe (precisely) how he/she can do it

https://medium.com/@tifa2up/image-classification-using-deep-neural-networks-a-beginner-friendly-approach-using-tensorflow-94b0a090ccd4

Features

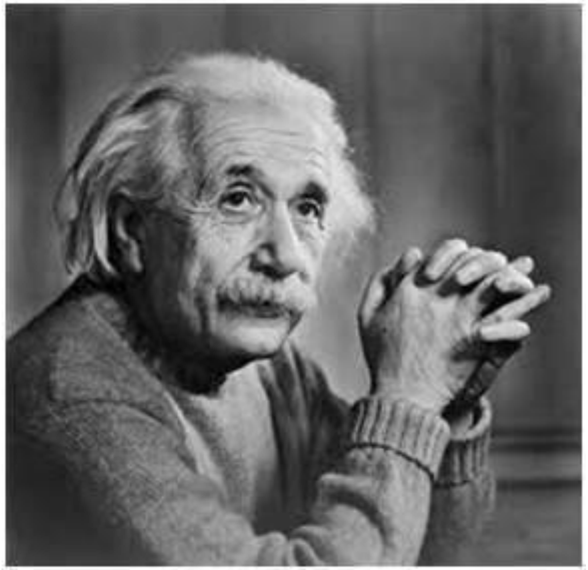


≈  + ... + Relative location

Features

- How to extract discriminative features?
  - Hand-crafted features: HOG, SIFT, SURF etc
  - Learned features: hidden states of DNN

Hand-crafted Features – Sobel Operator



-1	0	1
-2	0	2
-1	0	1

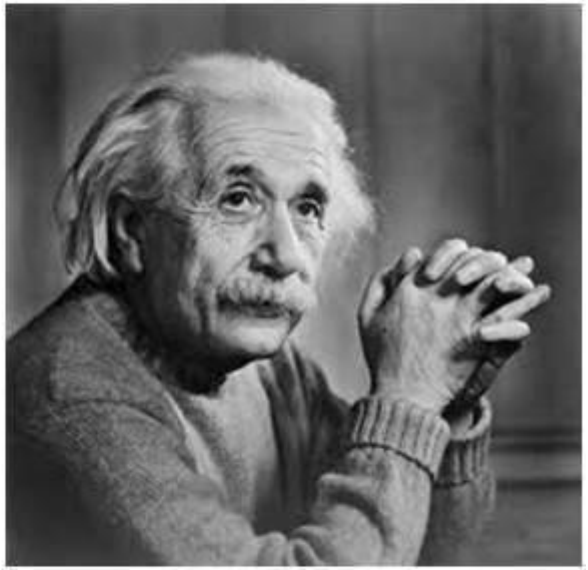
Vertical Mask

-1	-2	-1
0	0	0
1	2	1

Horizontal Mask

https://www.tutorialspoint.com/dip/sobel_operator.htm

Hand-crafted Features – Robinson Compass Mask



2	1	0
1	0	-1
0	-1	-2

South West Direction Mask

-2	-1	0
-1	0	1
0	1	2

North East Direction Mask

https://www.tutorialspoint.com/dip/robinson_compass_mask.htm

Hand-crafted Features – HOG

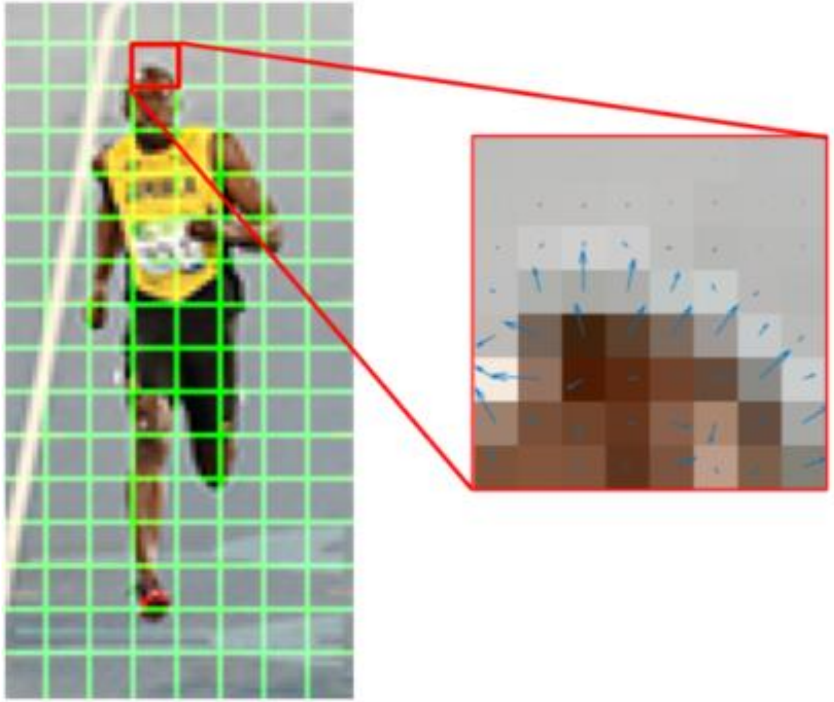
- Histograms of Oriented Gradients

-1	0	1

Vertical derivative

-1
0
1

Horizontal derivative



2	3	4	4	3	4	2	2
5	11	17	13	7	9	3	4
11	21	23	27	22	17	4	6
23	99	165	135	85	32	26	2
91	155	133	136	144	152	57	28
98	196	76	38	26	60	170	51
165	60	60	27	77	85	43	136
71	13	34	23	108	27	48	110

Gradient Magnitude

80	36	5	10	0	64	90	73
37	9	9	179	78	27	169	166
87	136	173	39	102	163	152	176
76	13	1	168	159	22	125	143
120	70	14	150	145	144	145	143
58	86	119	98	100	101	133	113
30	65	157	75	78	165	145	124
11	170	91	4	110	17	133	110

Gradient Direction

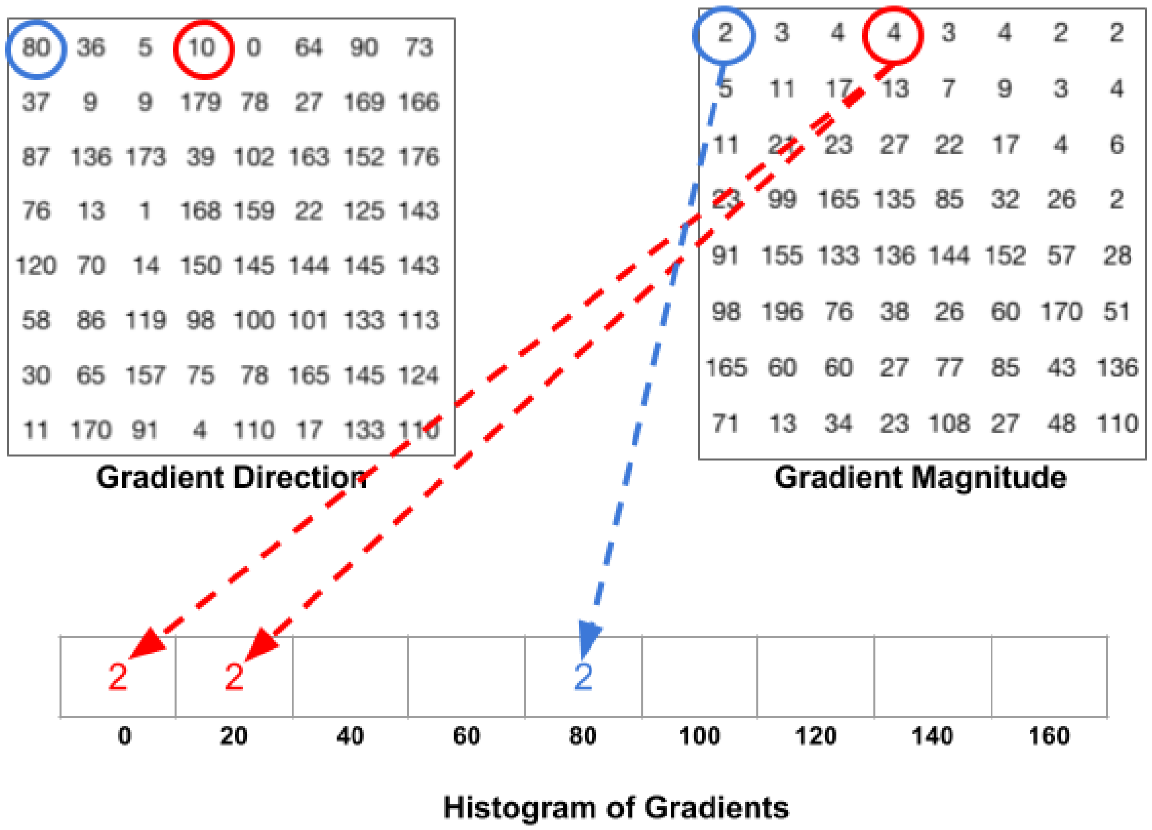
Center : The RGB patch and gradients represented using arrows. Right : The gradients in the same patch represented as numbers

https://www.learnopencv.com/histogram-of-oriented-gradients/

Dalal, Navneet, Triggs, et al. Histograms of Oriented Gradients for Human Detection. CVPR, 2005.

Hand-crafted Features – HOG

- Histograms of Oriented Gradients

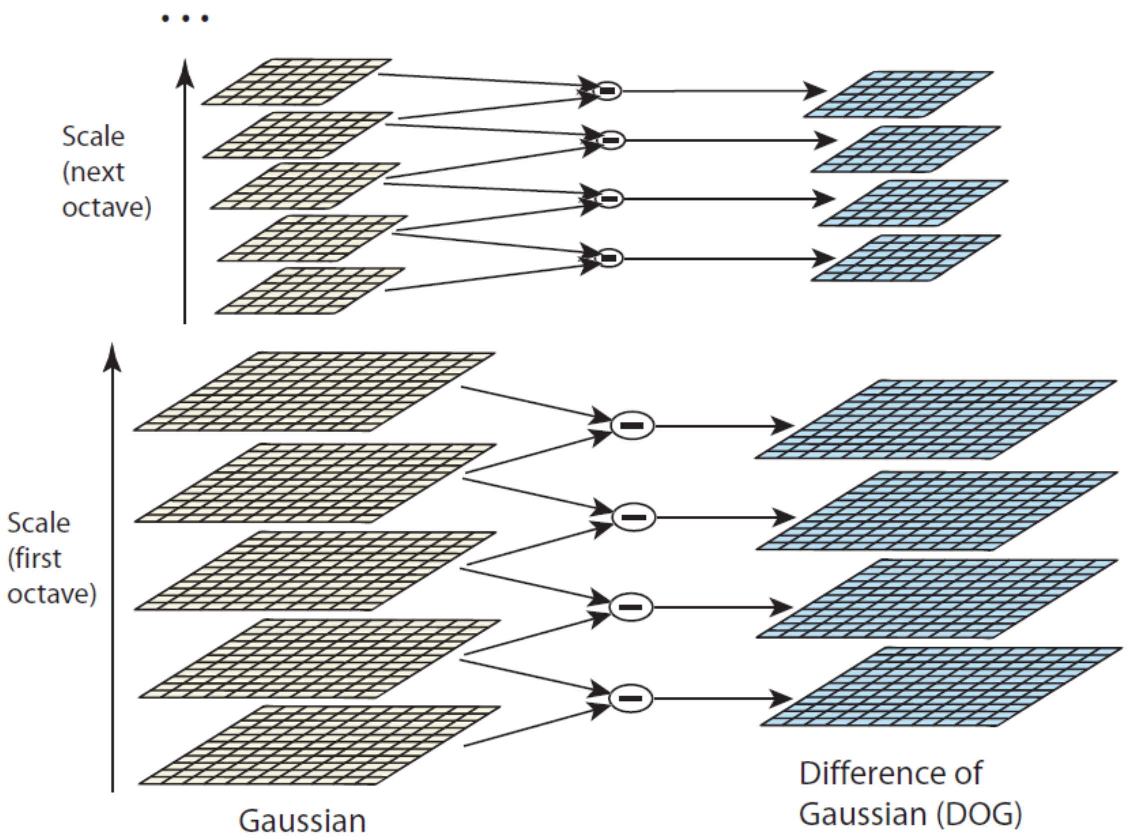


https://www.learnopencv.com/histogram-of-oriented-gradients/

Dalal, Navneet, Triggs, et al. Histograms of Oriented Gradients for Human Detection. CVPR, 2005.

Hand-crafted Features – SIFT

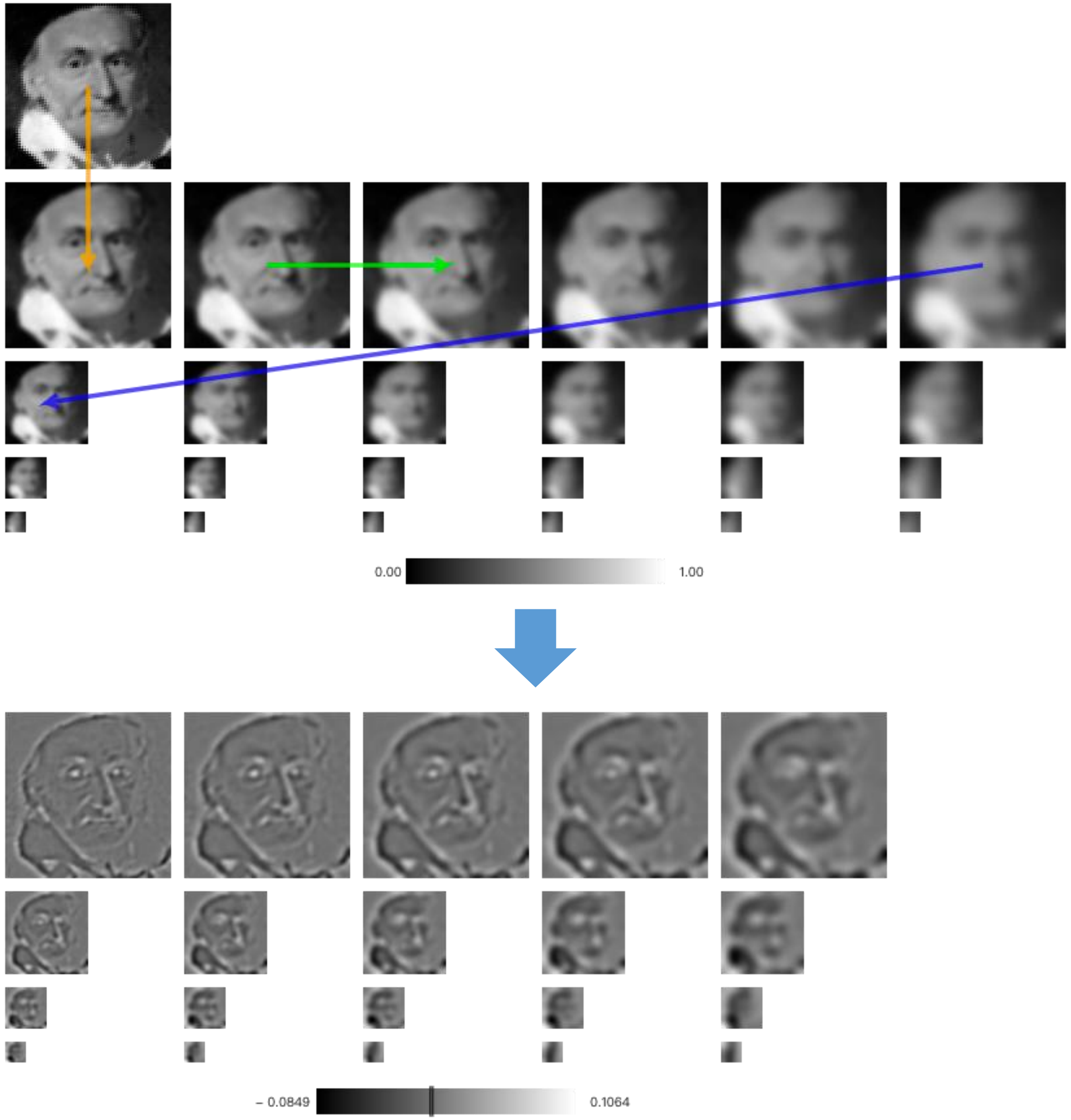
- Scale-Invariant Feature Transform

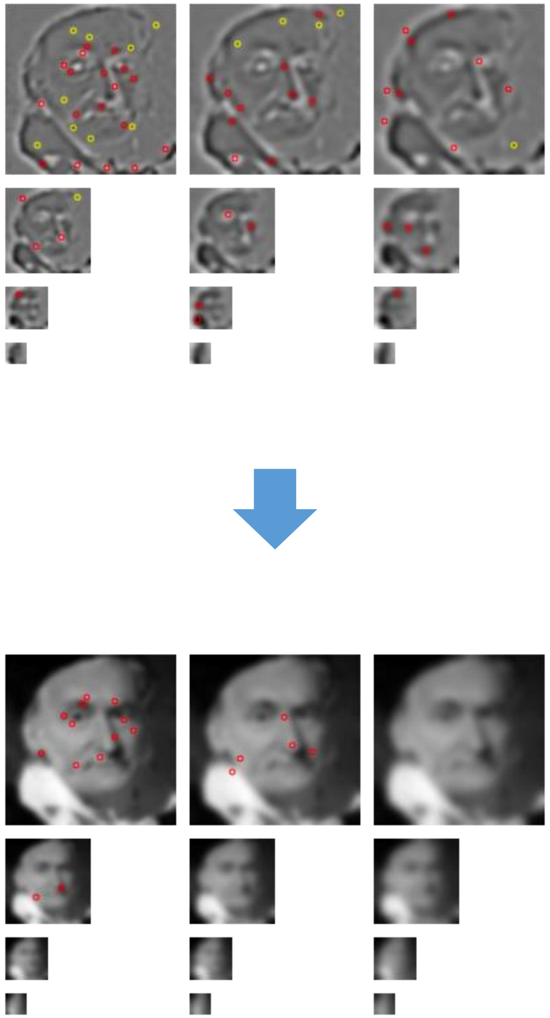


Lowe D G. Distinctive Image Features from Scale-Invariant Keypoints[C]// International Journal of Computer Vision. 2004:91-110.

http://weitz.de/sift/

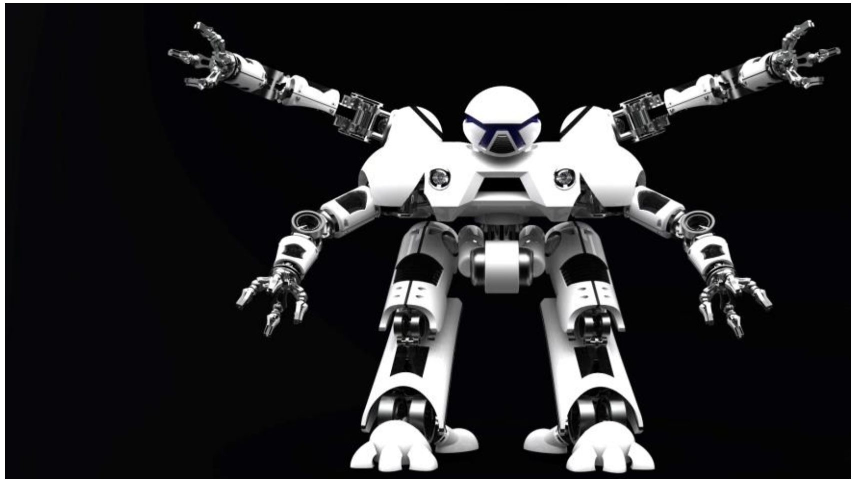
Hand-crafted Features – SIFT





http://weitz.de/sift/

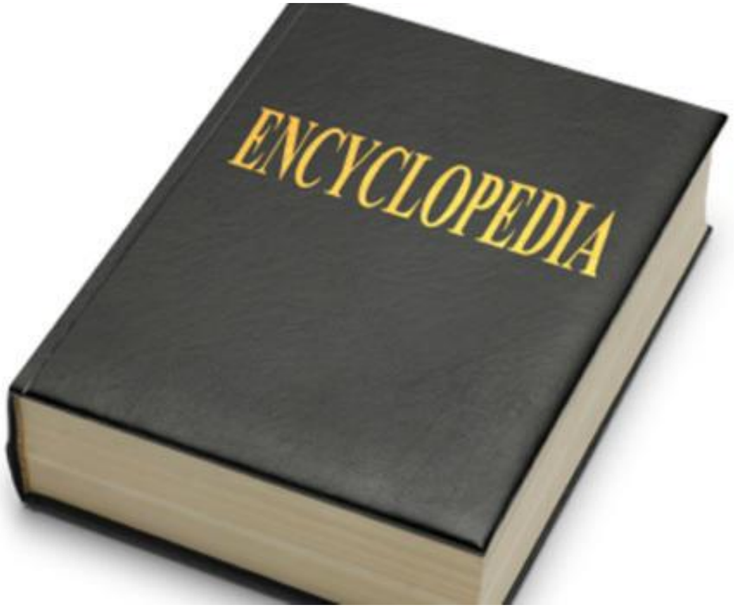
Hand-crafted Features vs Learned Features



- Hand-crafted Features
  - Challenging to design
  - Require expert domain knowledge
  - Not flexible

- Learned Features
  - Automatically learned by machines
  - No need of expert domain knowledge
  - Data-driven

Learned Features

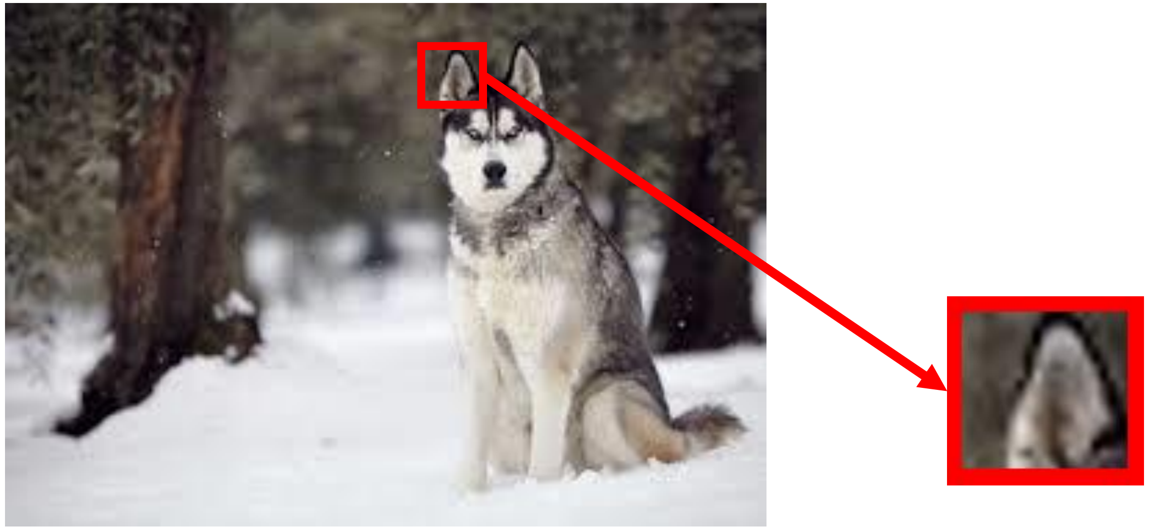


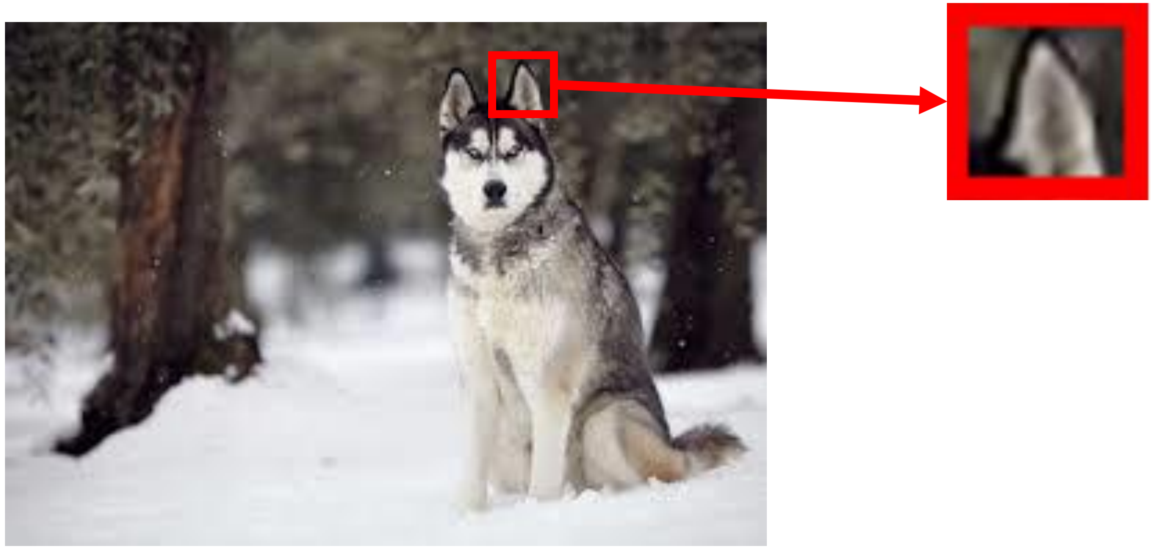
- We can design a new neural network
  - It can detect different patterns
  - It can detect similar patterns in different regions
  - It can roughly preserve the spatial information

Detect different patterns

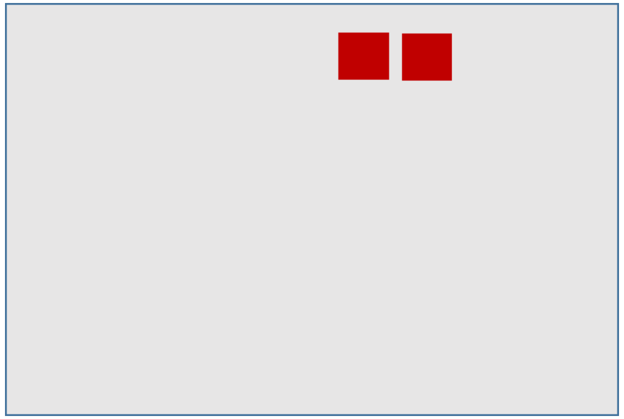
- Most patterns are much smaller in view of the whole image
  - A filter does not have to see the whole image to discover the pattern
    - One filter connects to small region with less parameters at a time

Detect the similar patterns in different regions

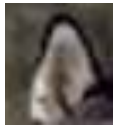


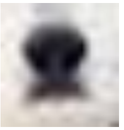


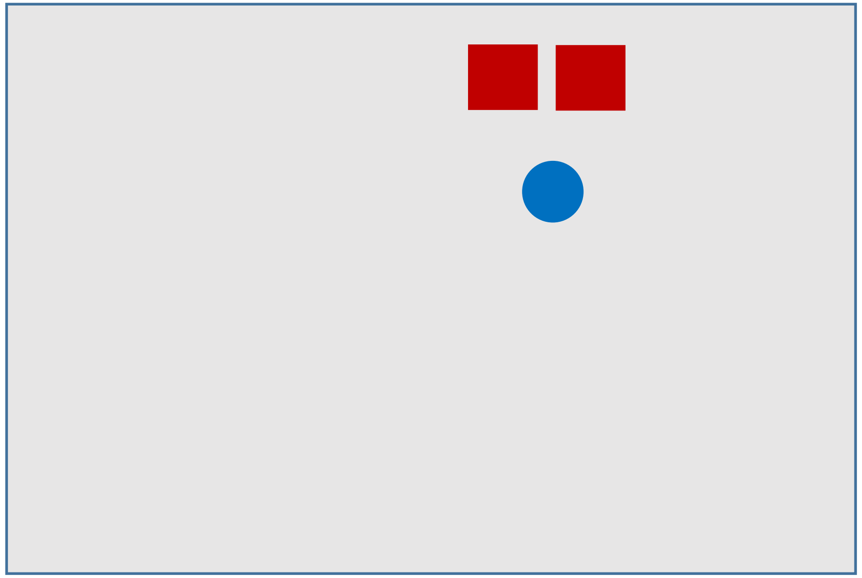
- The similar patterns appear in different regions
  - We can use one filter to detect similar patterns
    - One filter uses the same set of parameters for different regions



The spatial information

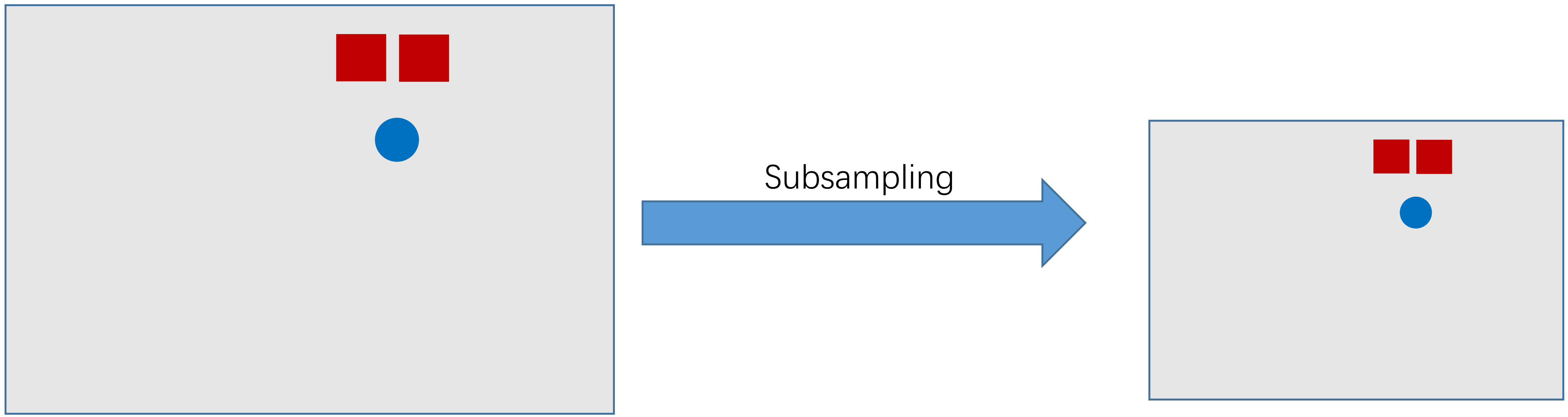






- For many vision tasks, the detected spatial information can be redundant

Roughly preserve the spatial information



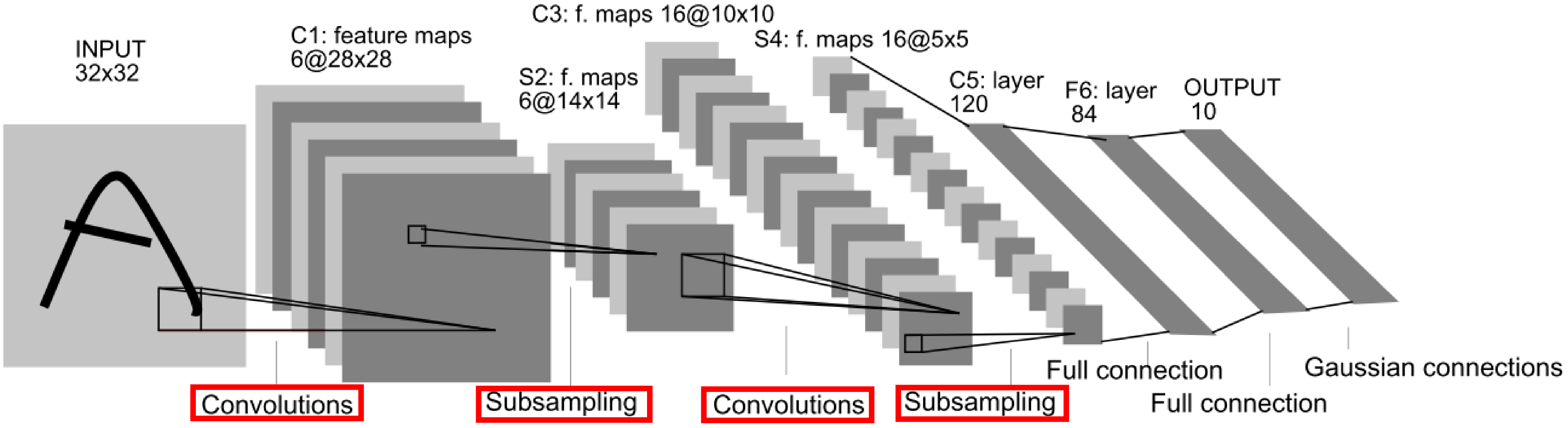
- The relative location of patterns will be preserved after subsampling
    - Using subsampling to lessen the parameters

Convolutional Neural Networks (CNN)

- Try to construct a new neural network
    - One filter connects to small region with less parameters at a time
    - One filter uses the same set of parameters for different regions
    - Using subsampling to lessen the parameters

Convolution

Pooling (Subsampling)

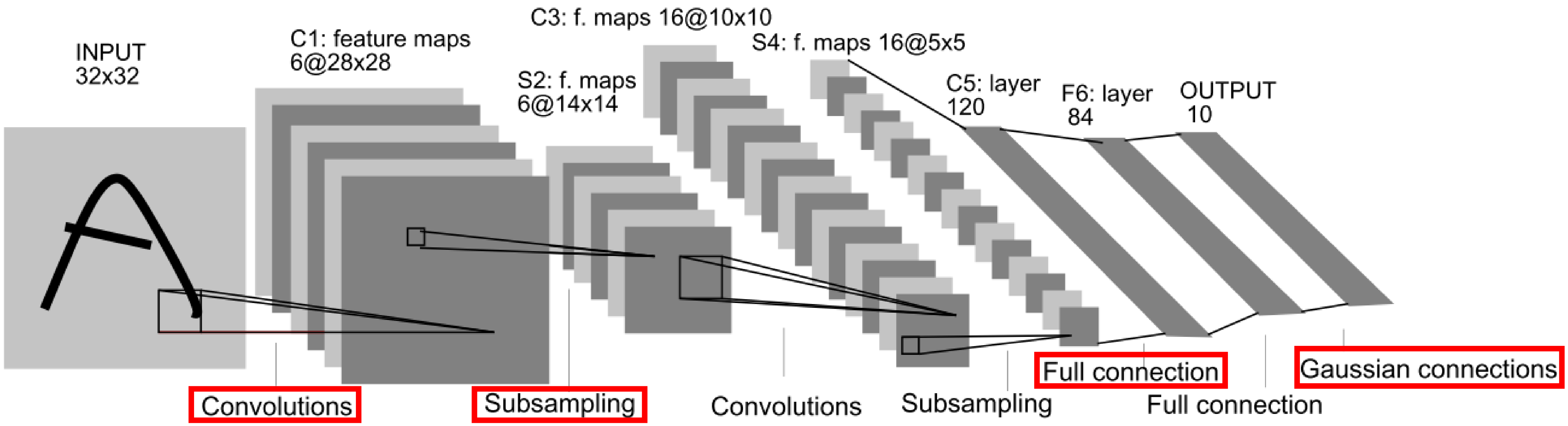


Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner, “Gradient-based learning applied to document recognition,” Proc. IEEE, vol. 86, no. 11, pp. 2278–2324, Feb. 1998.

Network Layers of CNN

How does CNN work

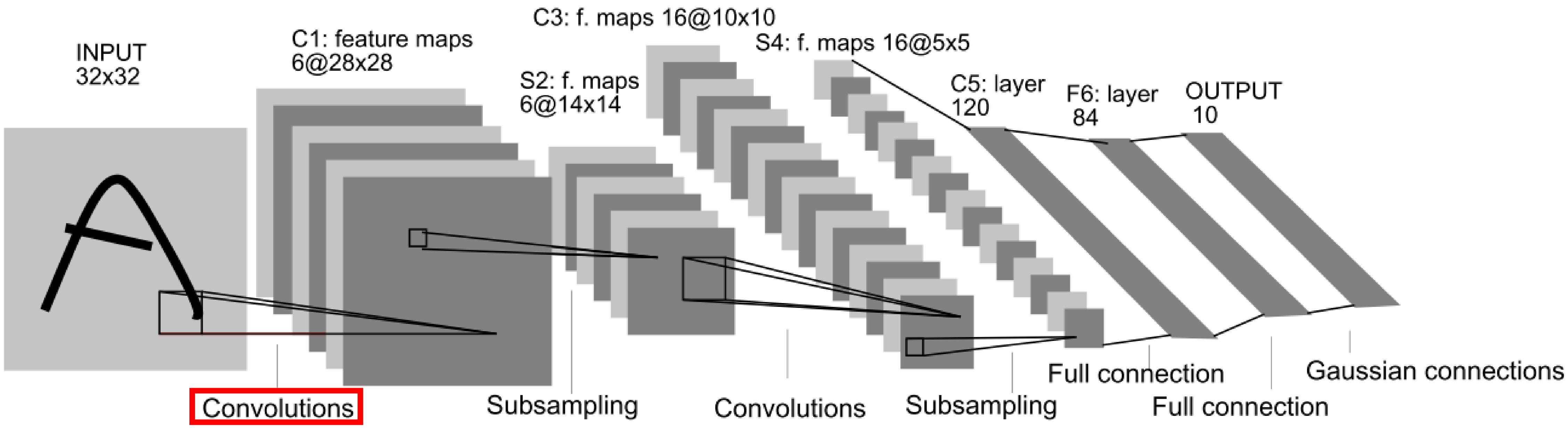
- A simple but important CNN – LeNet 5



- Given an input image, how to predict its label?

How does CNN work

- A simple but important CNN – LeNet 5

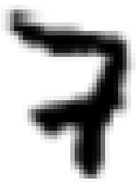


CNN – Convolution Layer

- **Input: Image**

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image



- **Filters**

Filter 1

1	1	1
-1	1	-1
1	-1	-1

Filter 2

1	-1	-1
-1	1	-1
-1	-1	1

Those are parameters to be learned!

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Filter 1

1	1	1
-1	1	-1
1	-1	-1

- One filter Connects to small region with less parameters at a time

$$1 \times 1 + 1 \times 1 + 1 \times 1$$
$$+0 \times (-1) + 0 \times 1 + 0 \times (-1)$$
$$+0 \times 1 + 1 \times (-1) + 1 \times (-1)$$

1

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 1

Filter 1

1	1	1
-1	1	-1
1	-1	-1

1 2

-1

- One filter uses the same set of parameters for different regions

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 3

Filter 1

1	1	1
-1	1	-1
1	-1	-1

1	5
3	

- One filter uses the same set of parameters for different regions

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 1

1	1	1
-1	1	-1
1	-1	-1

Filter 1

1	2	2	5
-1	-2	2	0
0	5	1	1
3	0	1	0

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 1

1	1	1
-1	1	-1
1	-1	-1

Filter 1

1	2	2	$\boxed{5}$
-1	-2	2	0
0	$\boxed{5}$	1	1
3	0	1	0

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 1

1	1	1
-1	1	-1
1	-1	-1

Filter 1

1	2	2	5
-1	-2	2	0
0	5	1	1
3	0	1	0

CNN – Convolution Layer

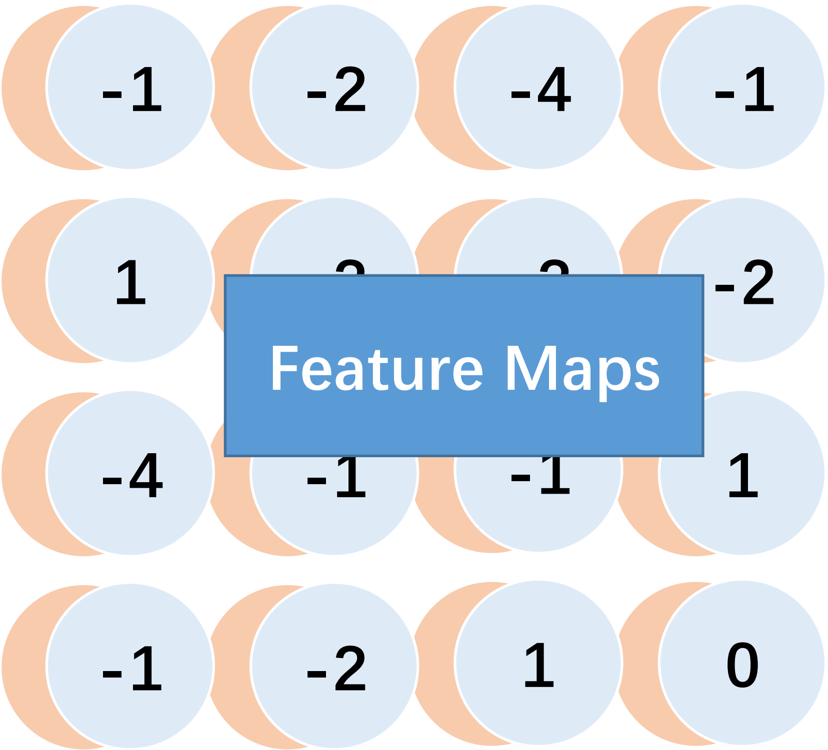
1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 1

1	-1	-1
-1	1	-1
-1	-1	1

Filter 2



$2 \times 4 \times 4$ Images

Ignore the bias and activation function!

CNN – Convolution Layer

1. How to deal with the color images?

2. How many parameters in the convolution layer?

3. What is the size of the feature maps?

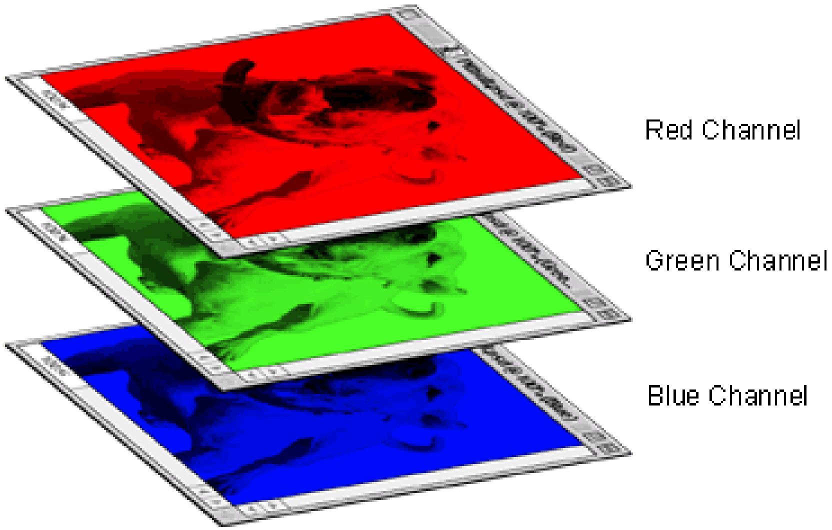
CNN – Convolution Layer

- How to deal with the color images?

1	1	1
-1	1	-1
1	-1	-1

1	-1	-1
-1	1	-1
1	-1	-1

colorful images



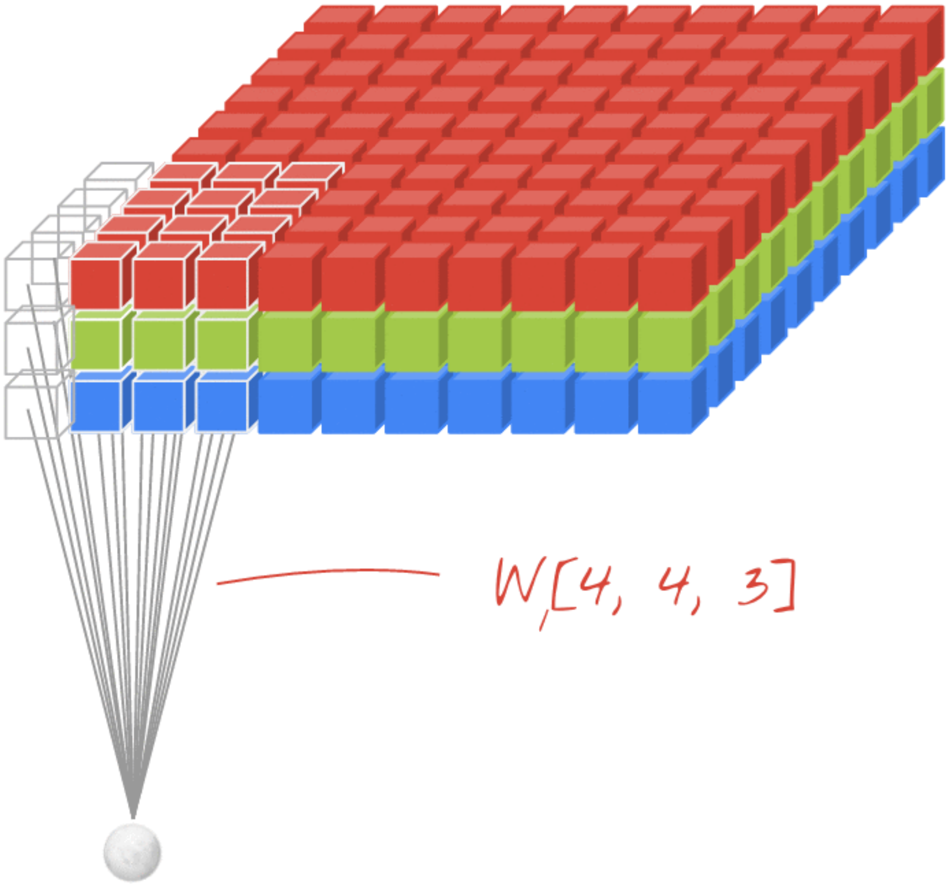
1	1	1	1	1	1
0	0	0	0	1	0
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

Feature maps is also a “colorful images”!

Image from: https://stackoverflow.com/questions/14113772/what-does-it-mean-to-change-the-color-channel

CNN – Convolution Layer

- How to deal with the colorful images?



https://sites.google.com/site/nttrungmtwiki/home/it/data-science---python/tensorflow/tensorflow-and-deep-learning-part-3?tmpl=%2Fsystem%2Fapp%2Ftemplates%2Fprint%2F&showPrintDialog=1

CNN – Convolution Layer

- How many parameters in the convolution layer?

1	1	1
-1	1	-1
1	-1	-1

Filter 1

$$9 = 3 \times 3$$

1	-1	-1
-1	1	-1
-1	-1	1

Filter 2

$$9 = 3 \times 3$$

......

-1	-1	-1
-1	1	-1
1	-1	1

Filter N

$$9 = 3 \times 3$$

There are $9N$ parameters

The power of sharing weights!

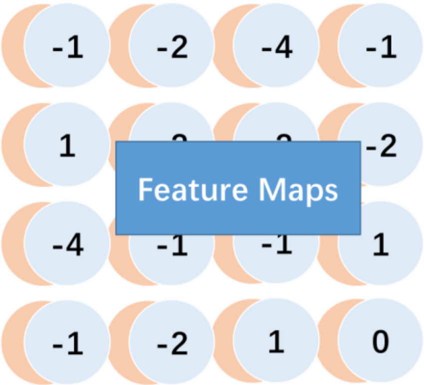
CNN – Convolution Layer

- What is the size of the feature maps?

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

2 filters
Filter size: 3×3
Stride:1



$2 \times 4 \times 4$ image

Input image: $C \times H \times W$

Filters: $N \times n \times n$

Stride: s

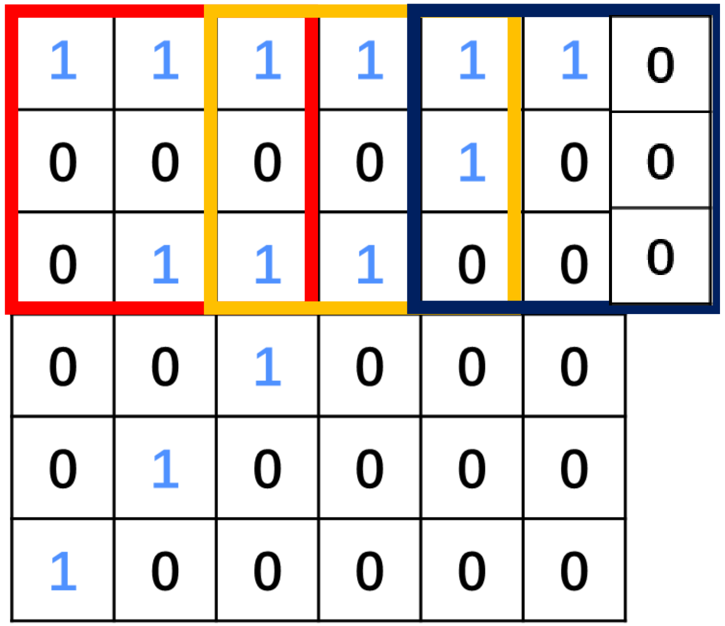
Feature maps=? $N \times \left(\frac{H-n}{s}+1\right) \times \left(\frac{W-n}{s}+1\right)$

May not divisible!

CNN – Convolution Layer

- Padding

Stride = 2



6 × 6 image

1	1	1
-1	1	-1
1	-1	-1

- Two ways

  1. Ignore the extra pixels

  2. Fill with zero

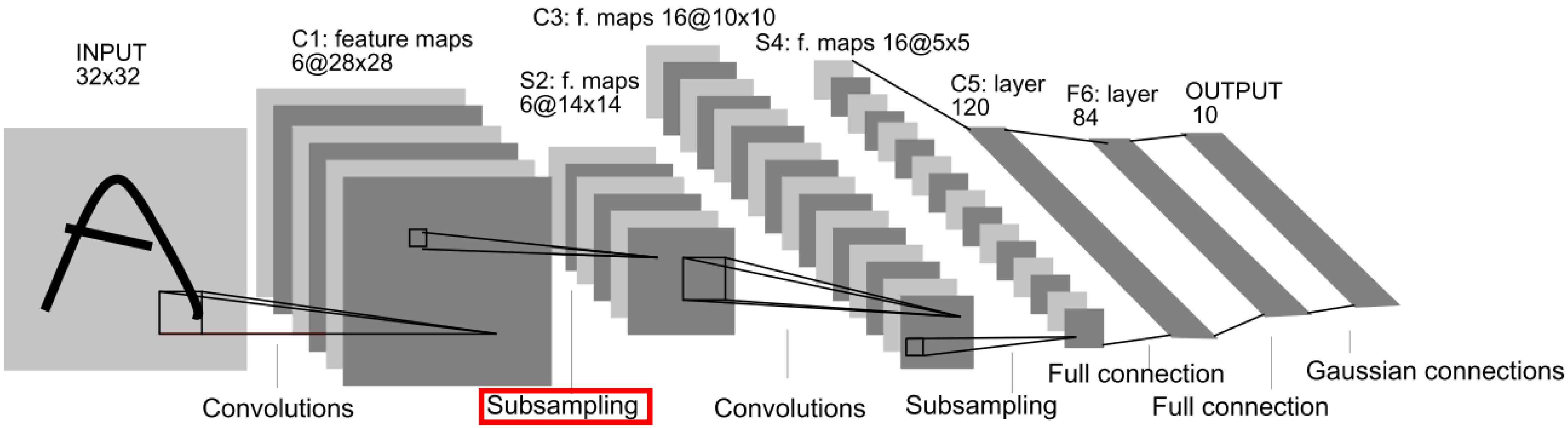
     Add P_H and P_W zeros

Feature maps = $N \times \left(\frac{H-n+P_H}{s}+1\right) \times \left(\frac{W-n+P_W}{s}+1\right)$

参考 cs231n 2017 lecture5, page62和 https://www.tensorflow.org/api_guides/python/nn#Notes_on_SAME_Convolution_Padding

CNN – Pooling Layer

- LeNet 5



CNN – Pooling Layer

- The output from the convolution layer can be huge

$$\left.\begin{array}{l}\textbf{Input image: } 1 \times 96 \times 96 \\ \\ \textbf{Filters: } 400 \times 8 \times 8 \\ \\ \textbf{Stride: } 1\end{array}\right\} \textbf{Feature maps=} 400 \times (96 - 8 + 1) \times (96 - 8 + 1)$$

- Output of the convolution layer

$$3168400 = 400 \times (96 - 8 + 1) \times (96 - 8 + 1)$$

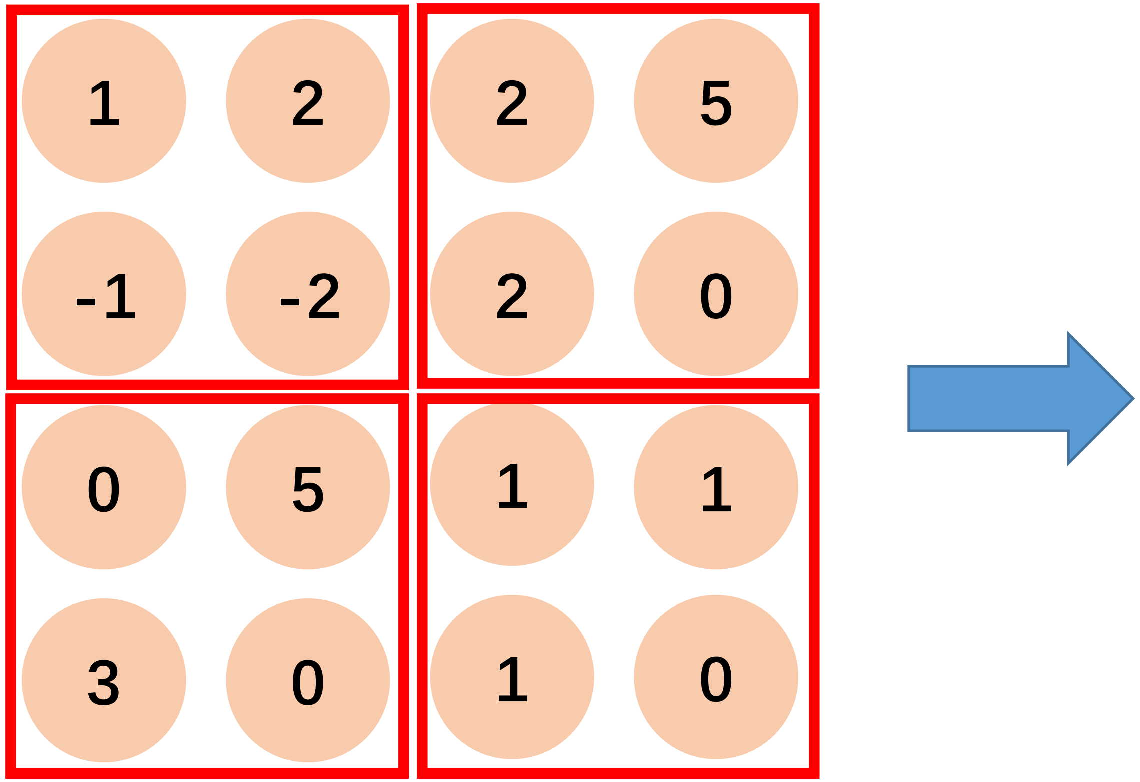
  - Hard to train
  - Overfitting

CNN – Pooling Layer

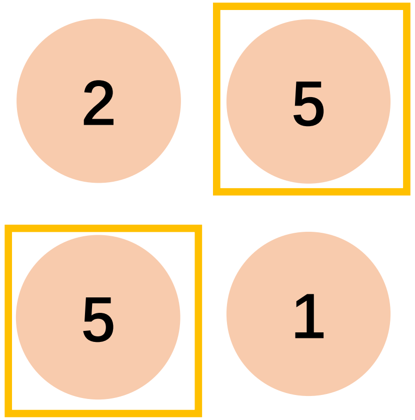
- Max Pooling

1	1	1
-1	1	-1
1	-1	-1

Filter 1



Convolved Feature



Pooled Feature

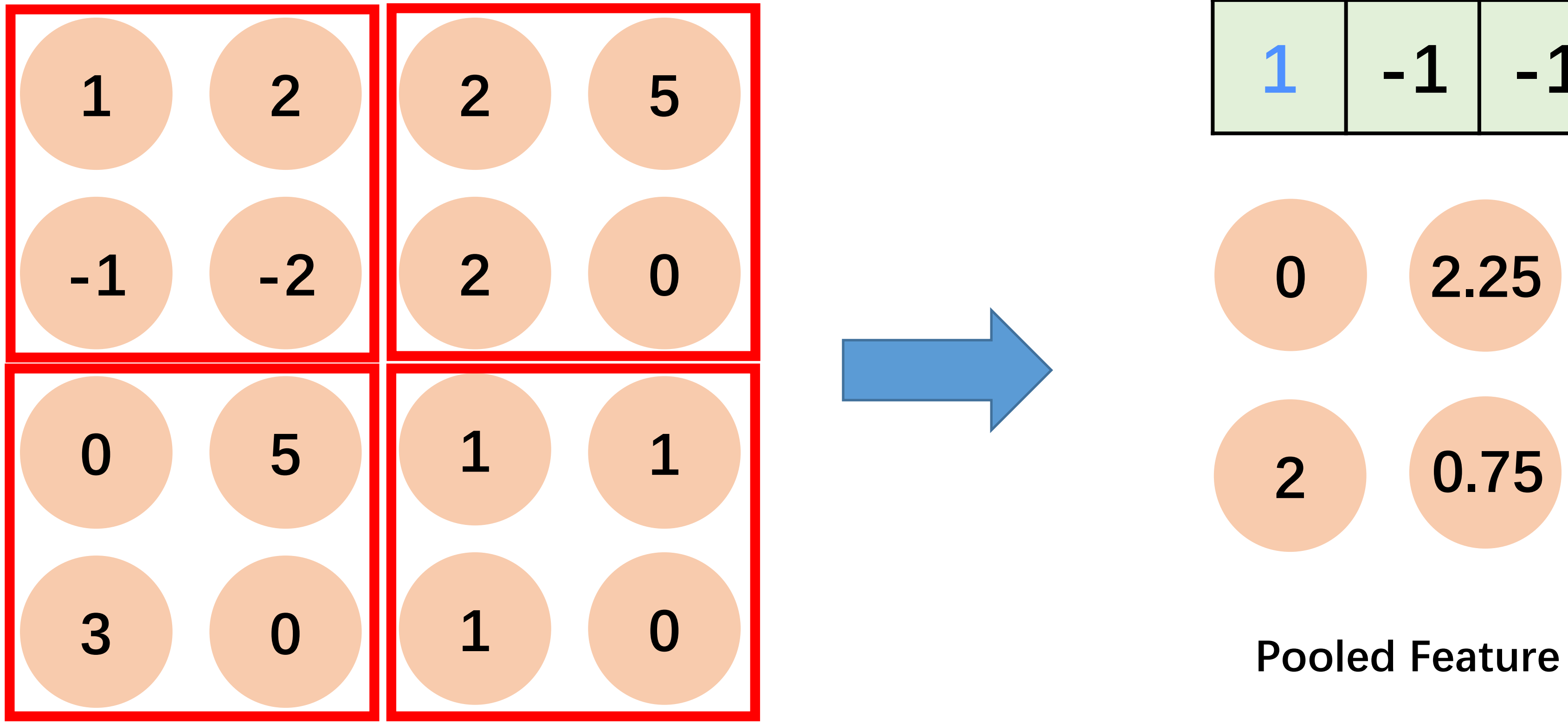
➢ Using subsampling to lessen the parameters

CNN – Pooling Layer

- Average Pooling

Filter 1

1	1	1
-1	1	-1
1	-1	-1

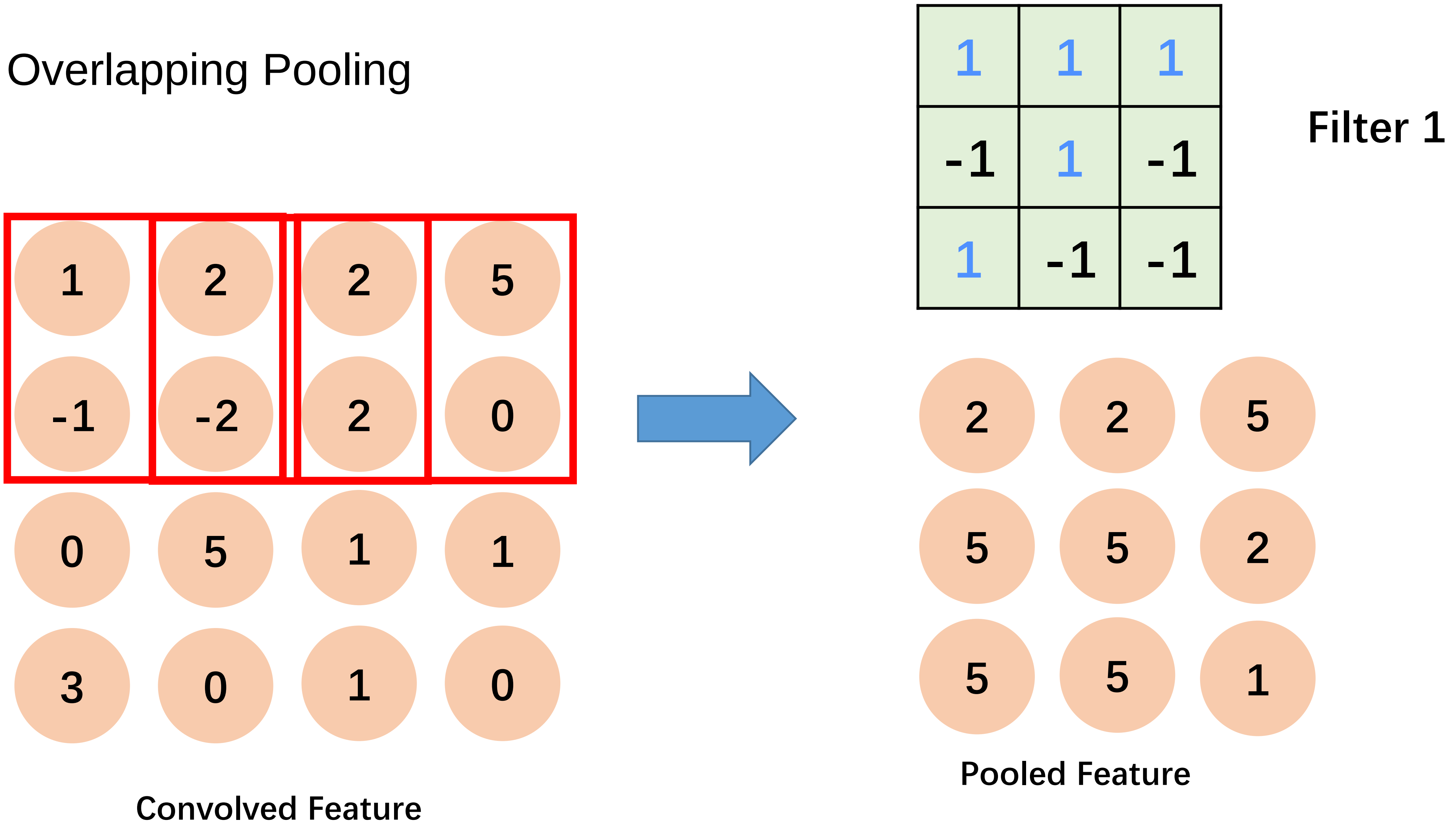


Convolved Feature

Pooled Feature

CNN – Pooling Layer

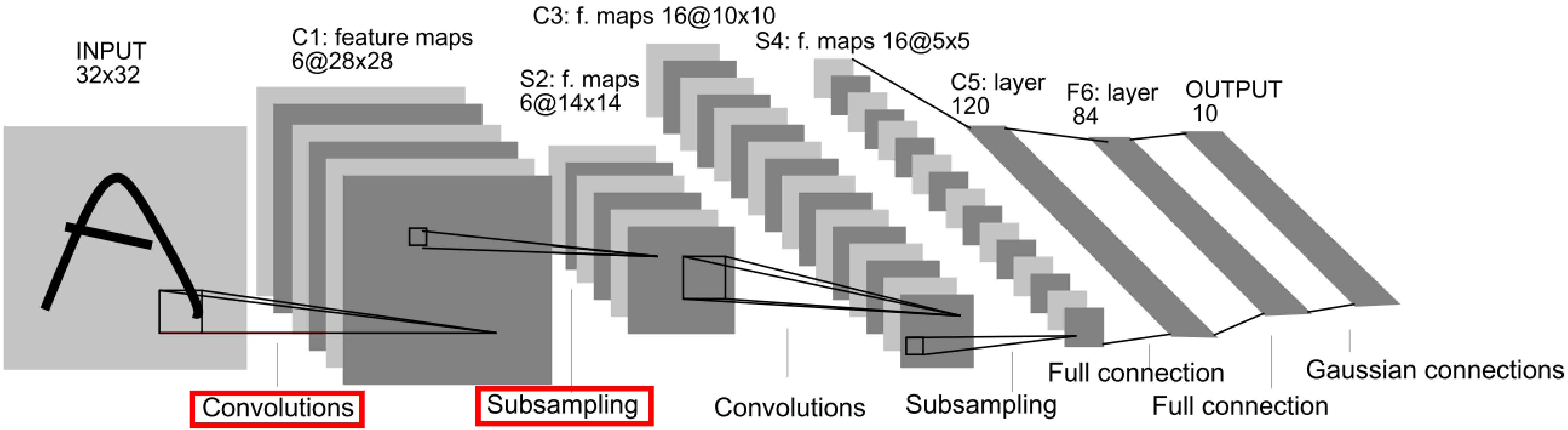
- Overlapping Pooling



[1] A. Krizhevsky, I. Sutskever, and G. E. Hinton, “ImageNet Classification with Deep Convolutional Neural Networks,” *NIPS*, 2012.

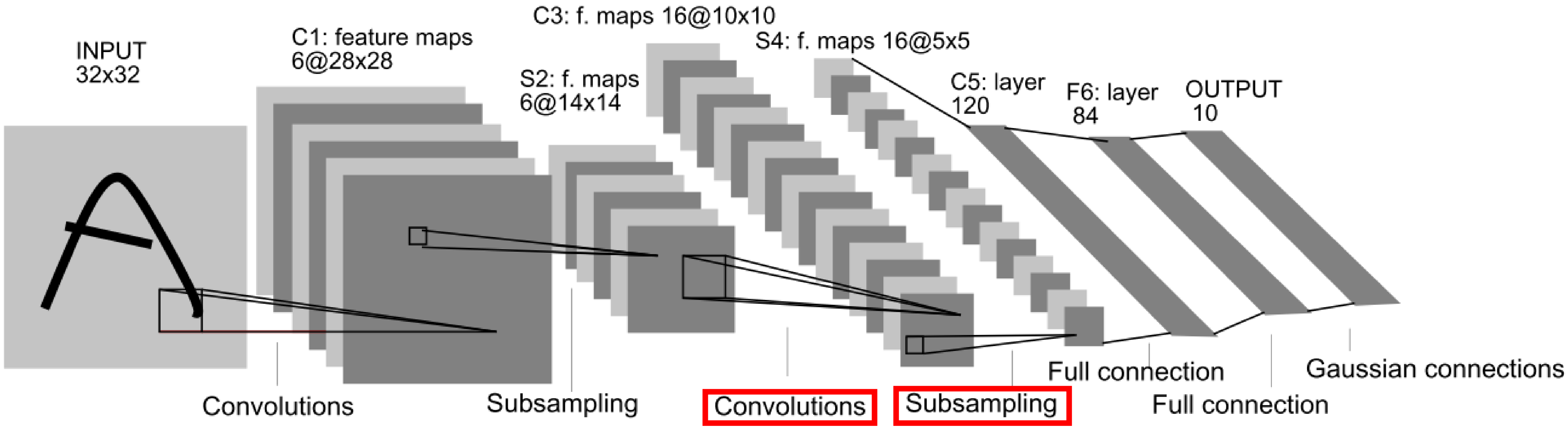
CNN - Convolution + Pooling

- LeNet 5



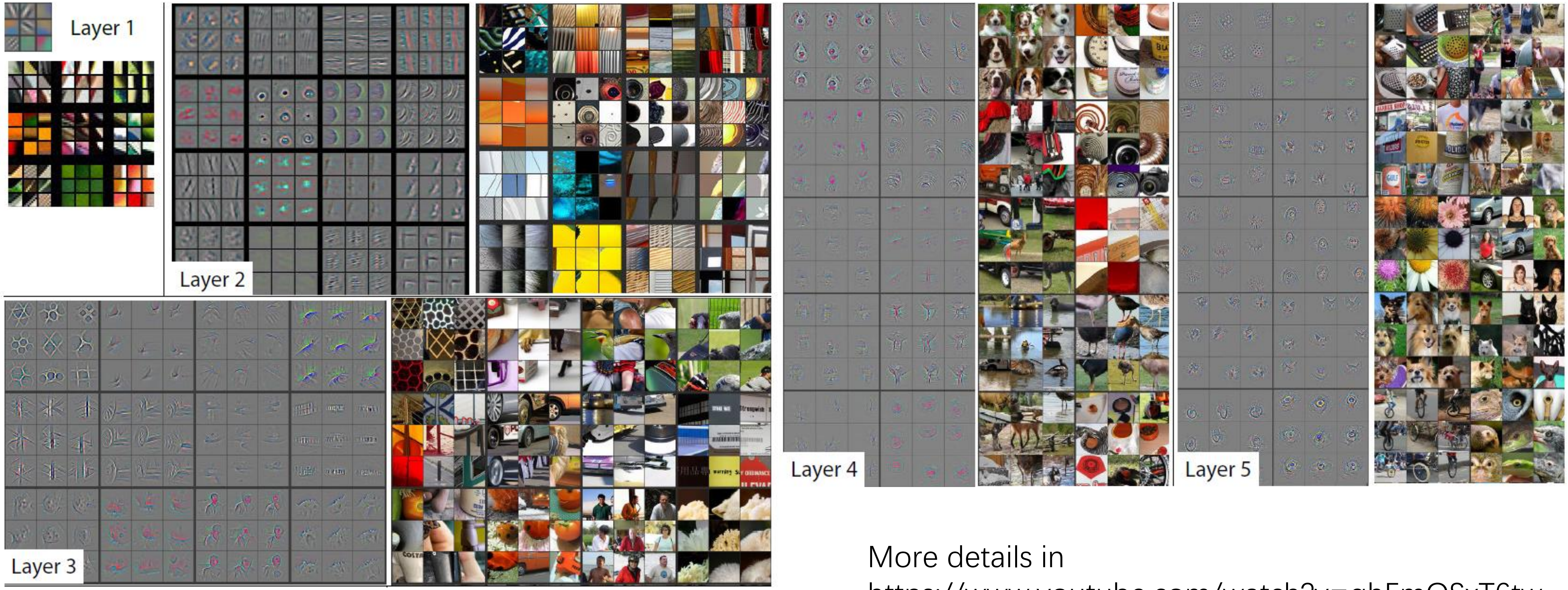
CNN - Convolution + Pooling

- LeNet 5



- Why more convolution and pooling layer?

Learned Features by CNN



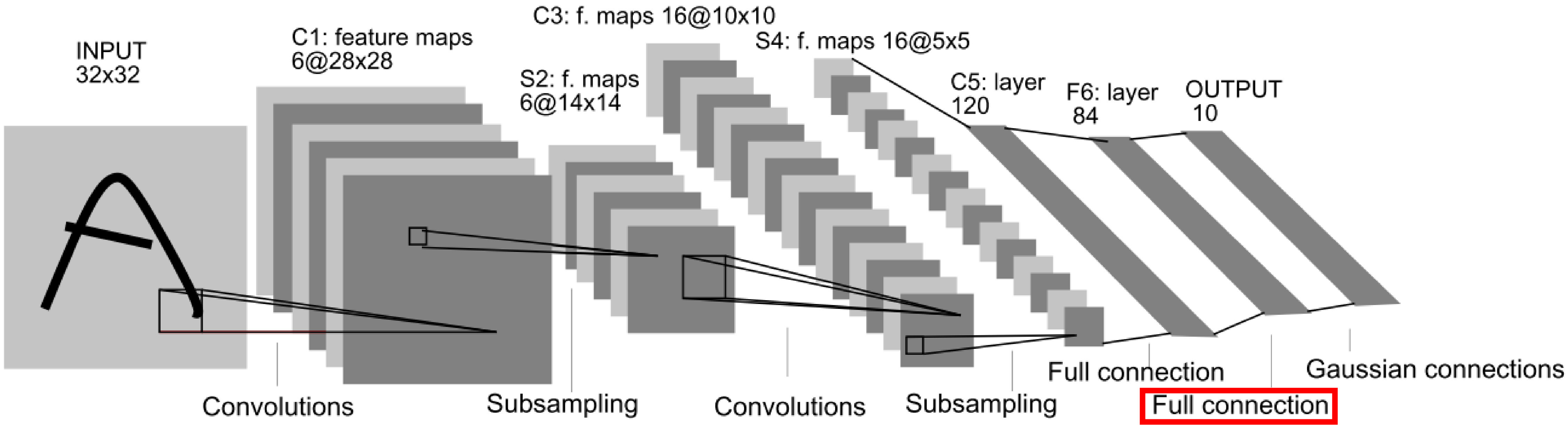
More details in
https://www.youtube.com/watch?v=ghEmQSxT6tw

- Deeper layers, more specific features

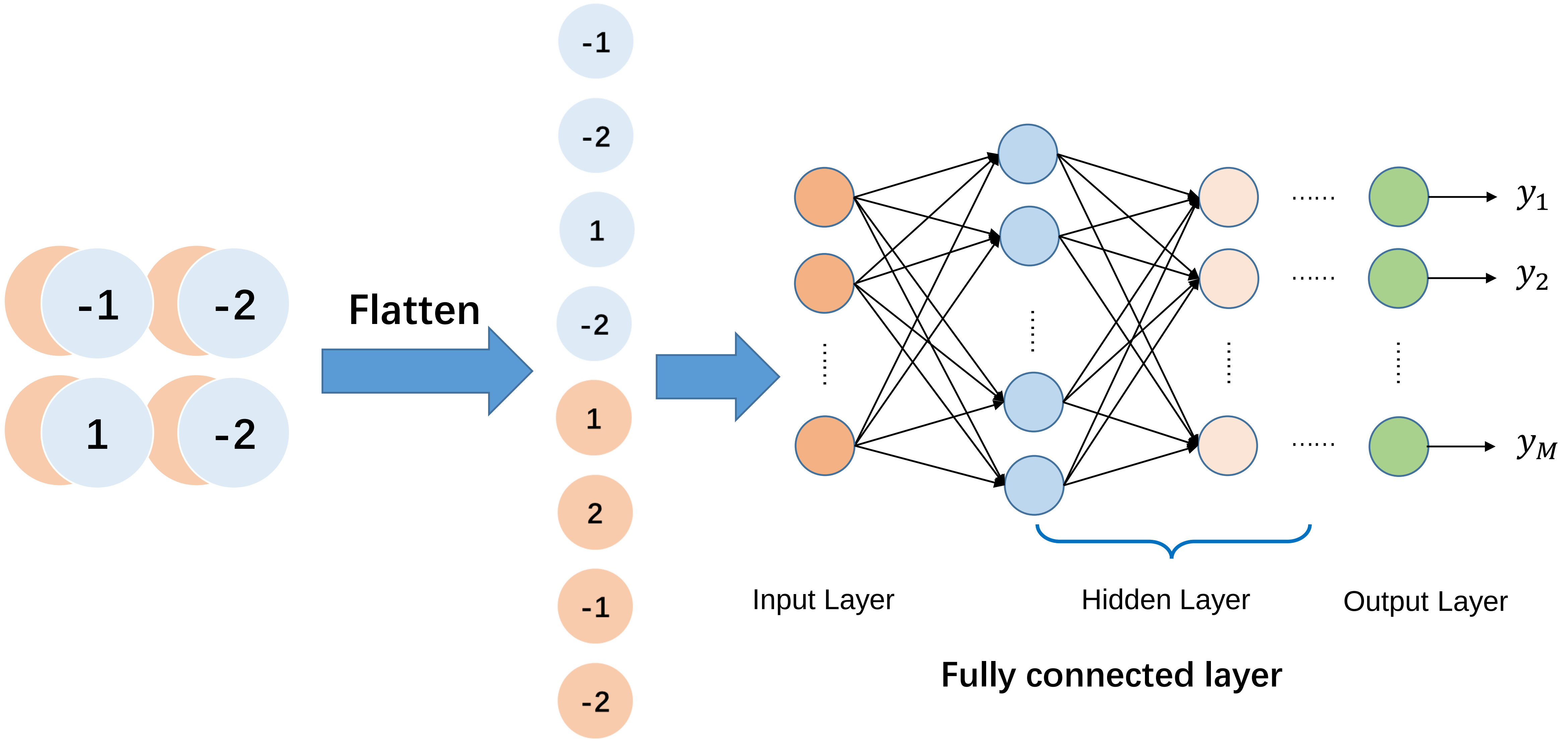
Zeiler M D, Fergus R. Visualizing and Understanding Convolutional Networks[J]. 2013, 8689:818-833.

CNN – Fully connected layer

- LeNet 5

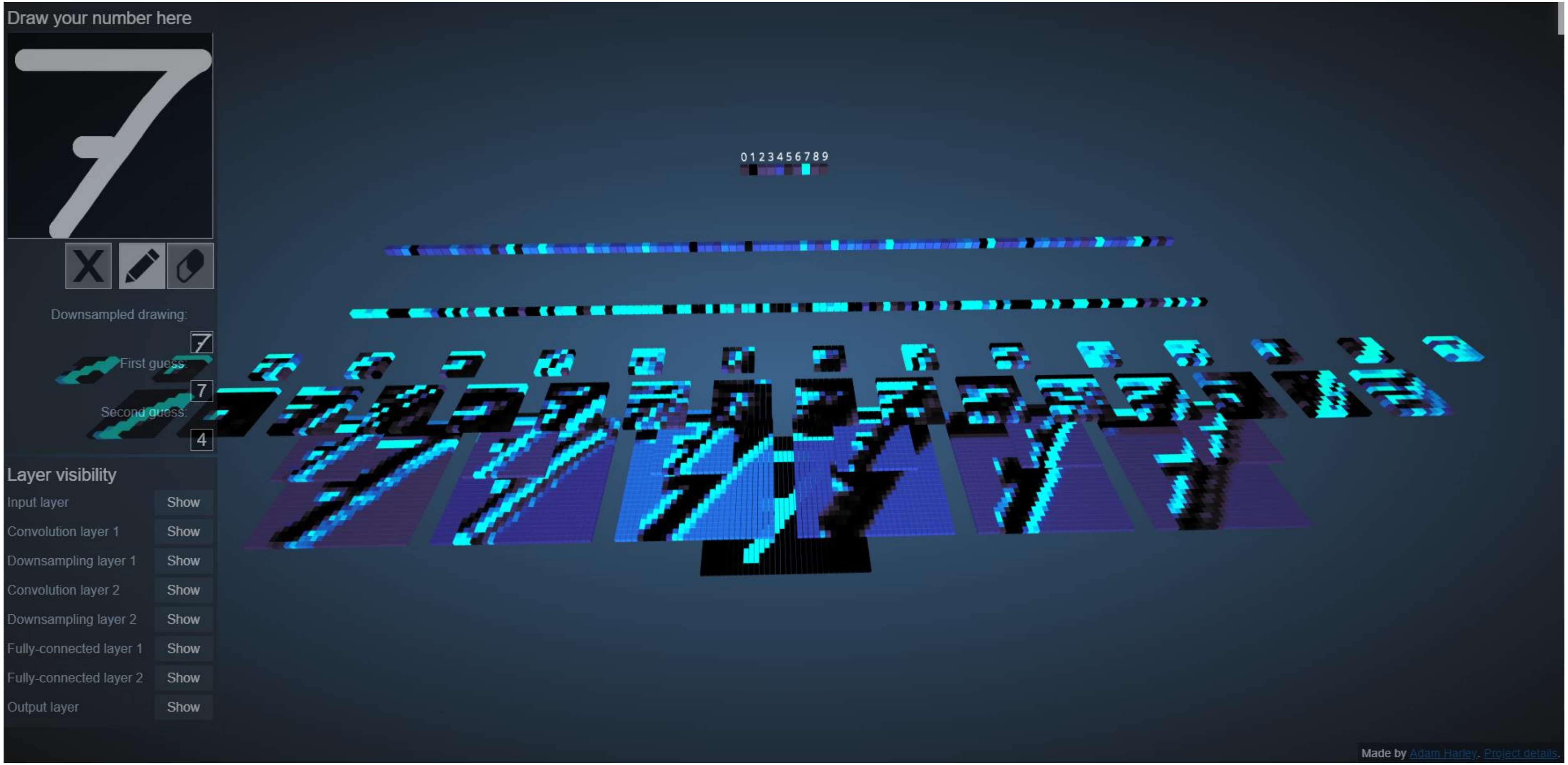


CNN – Fully connected layer



Fully connected layer

CNN – Visualization



3D convolutional network visualization http://scs.ryerson.ca/~aharley/vis/conv/

A. W. Harley, "An Interactive Node-Link Visualization of Convolutional Neural Networks," in ISVC, pages 867-877, 2015

CNN – Hyperparameters

- Convolution layers
  - Number of filters
  - Size of filters
  - Stride
- Pooling layers
  - Window size
  - Window stride
- Fully connected layers
  - Number of layers
  - Number of neurons

CNN – Traditional Methods

- Comparison

	Traditional Methods	CNN
Filters	Manually design	Learn automatically
Layers	Few	Can be quite some
Features	Low level features	From low to high level features

CNN is more powerful!

Learning a CNN

Loss functions

- Mean squared error (MSE)

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

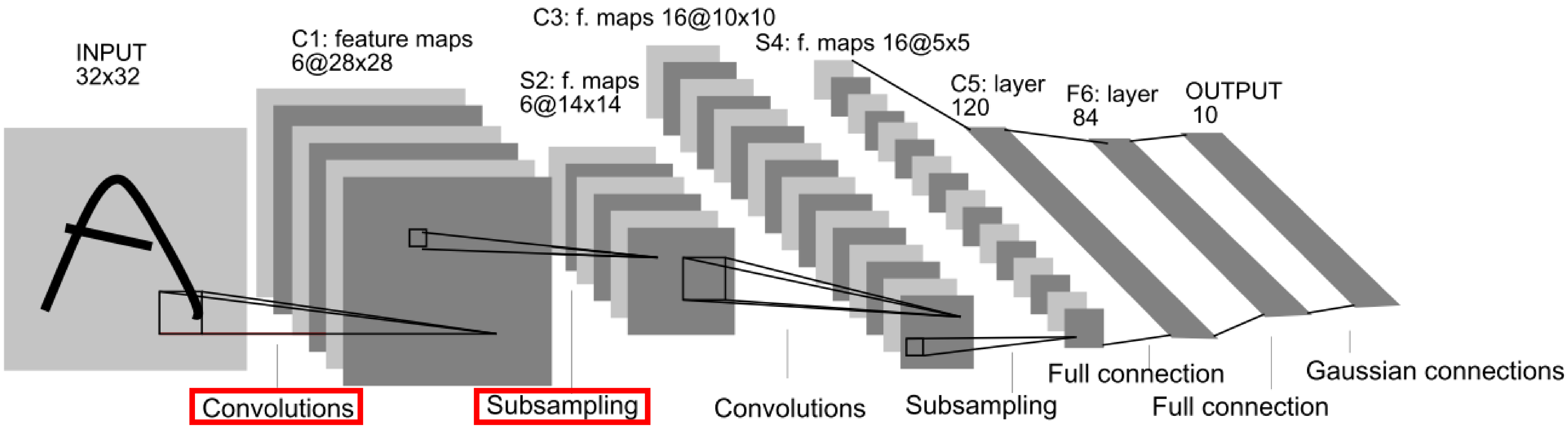
- Cross entropy loss

$$\text{Loss} = -\sum_{i=1}^{n} Y_i \log p_i$$

- User defined loss

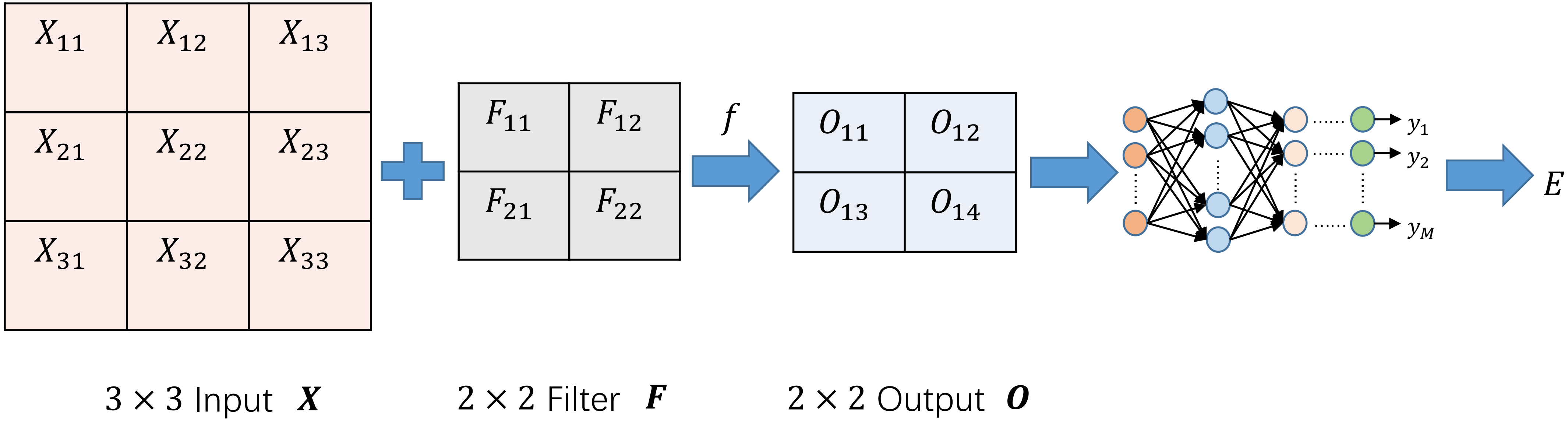
Backpropagation

- A simple but important CNN – LeNet 5



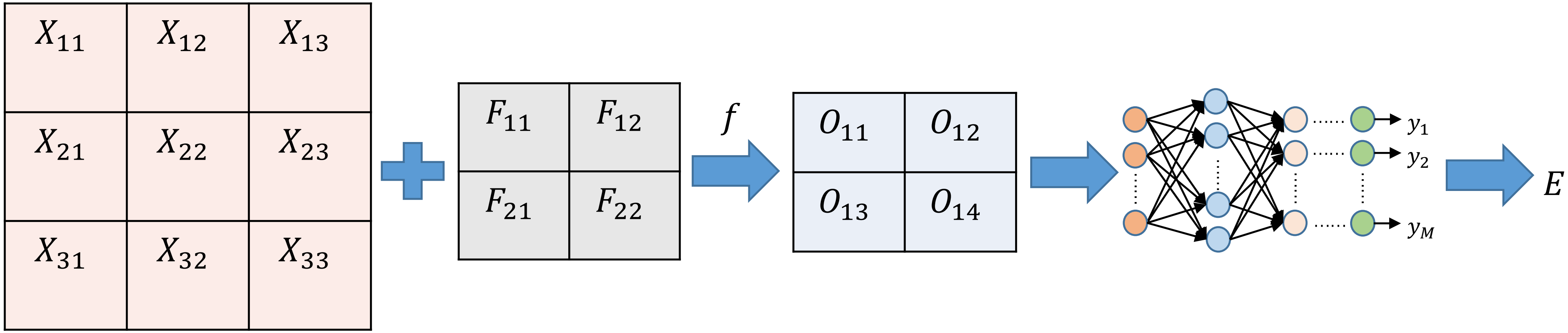
- Back-propagation – Chain rule
- How to compute the gradients of convolution layers and pooling layers?

Backpropagation – Convolution Layer



- The output of convolution operation $\boldsymbol{O} = f(\boldsymbol{F}, \boldsymbol{X})$
- The loss E
- Assume that we have already computed $\frac{\partial E}{\partial O_{ij}}$, and of course all partial derivatives of latter layers

Backpropagation – Convolution Layer



- How to compute $\frac{\partial E}{\partial F_{11}}$?

Chain Rule

$$\frac{\partial E}{\partial F_{11}} = \left(\frac{\partial E}{\partial \boldsymbol{O}}\right)^T \frac{\partial \boldsymbol{O}}{\partial F_{11}}$$

$$\frac{\partial E}{\partial \boldsymbol{O}} = \nabla E(O_{11}, O_{12}, O_{21}, O_{22})$$

$$= \left(\frac{\partial E}{\partial O_{11}}, \frac{\partial E}{\partial O_{12}}, \frac{\partial E}{\partial O_{21}}, \frac{\partial E}{\partial O_{22}}\right)^T$$

$$\boldsymbol{O} = f(\boldsymbol{F}, \boldsymbol{X}) = (O_{11}, O_{12}, O_{21}, O_{22})^T \triangleq \left(f_1(\boldsymbol{F}, \boldsymbol{X}), f_2(\boldsymbol{F}, \boldsymbol{X}), f_3(\boldsymbol{F}, \boldsymbol{X}), f_4(\boldsymbol{F}, \boldsymbol{X})\right)^T$$

$$\frac{\partial \boldsymbol{O}}{\partial F_{11}} = \left(\frac{\partial f_1}{\partial F_{11}}, \frac{\partial f_2}{\partial F_{11}}, \frac{\partial f_3}{\partial F_{11}}, \frac{\partial f_4}{\partial F_{11}}\right)^T$$

Backpropagation – Convolution Layer

$$\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}}\frac{\partial f_1}{\partial F_{11}} + \frac{\partial E}{\partial O_{12}}\frac{\partial f_2}{\partial F_{11}} + \frac{\partial E}{\partial O_{21}}\frac{\partial f_3}{\partial F_{11}} + \frac{\partial E}{\partial O_{22}}\frac{\partial f_4}{\partial F_{11}}$$

$$\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}}\frac{\partial f_1}{\partial F_{12}} + \frac{\partial E}{\partial O_{12}}\frac{\partial f_2}{\partial F_{12}} + \frac{\partial E}{\partial O_{21}}\frac{\partial f_3}{\partial F_{12}} + \frac{\partial E}{\partial O_{22}}\frac{\partial f_4}{\partial F_{12}}$$

$$\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}}\frac{\partial f_1}{\partial F_{21}} + \frac{\partial E}{\partial O_{12}}\frac{\partial f_2}{\partial F_{21}} + \frac{\partial E}{\partial O_{21}}\frac{\partial f_3}{\partial F_{21}} + \frac{\partial E}{\partial O_{22}}\frac{\partial f_4}{\partial F_{21}}$$

$$\frac{\partial E}{\partial F_{22}} = \frac{\partial E}{\partial O_{11}}\frac{\partial f_1}{\partial F_{22}} + \frac{\partial E}{\partial O_{12}}\frac{\partial f_2}{\partial F_{22}} + \frac{\partial E}{\partial O_{21}}\frac{\partial f_3}{\partial F_{22}} + \frac{\partial E}{\partial O_{22}}\frac{\partial f_4}{\partial F_{22}}$$

Backpropagation – Convolution Layer

$$\boldsymbol{O} = f(\boldsymbol{F};\boldsymbol{X})$$

$$f_1(\boldsymbol{X}) = O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$f_2(\boldsymbol{X}) = O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$f_3(\boldsymbol{X}) = O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$f_4(\boldsymbol{X}) = O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$

$$\frac{\partial f_1}{\partial F_{11}} = X_{11} \qquad \frac{\partial f_2}{\partial F_{11}} = X_{12} \qquad \frac{\partial f_3}{\partial F_{11}} = X_{21} \qquad \frac{\partial f_4}{\partial F_{11}} = X_{22}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\frac{\partial f_1}{\partial F_{22}} = X_{22} \qquad \frac{\partial f_2}{\partial F_{22}} = X_{23} \qquad \frac{\partial f_3}{\partial F_{22}} = X_{32} \qquad \frac{\partial f_4}{\partial F_{22}} = X_{33}$$

Backpropagation – Convolution Layer

$$\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$$

$$\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} X_{12} + \frac{\partial E}{\partial O_{12}} X_{13} + \frac{\partial E}{\partial O_{21}} X_{22} + \frac{\partial E}{\partial O_{22}} X_{23}$$

$$\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}} X_{21} + \frac{\partial E}{\partial O_{12}} X_{22} + \frac{\partial E}{\partial O_{21}} X_{31} + \frac{\partial E}{\partial O_{22}} X_{32}$$

$$\frac{\partial E}{\partial F_{22}} = \frac{\partial E}{\partial O_{11}} X_{22} + \frac{\partial E}{\partial O_{12}} X_{23} + \frac{\partial E}{\partial O_{21}} X_{32} + \frac{\partial E}{\partial O_{22}} X_{33}$$

Backpropagation – Convolution Layer

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} + \begin{bmatrix} \frac{\partial E}{\partial O_{11}} & \frac{\partial E}{\partial O_{12}} \\ \frac{\partial E}{\partial O_{21}} & \frac{\partial E}{\partial O_{22}} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\partial E}{\partial F_{11}} & \frac{\partial E}{\partial F_{12}} \\ \frac{\partial E}{\partial F_{21}} & \frac{\partial E}{\partial F_{22}} \end{bmatrix}$$

$$\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$$

$$\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} X_{12} + \frac{\partial E}{\partial O_{12}} X_{13} + \frac{\partial E}{\partial O_{21}} X_{22} + \frac{\partial E}{\partial O_{22}} X_{23}$$

$$\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}} X_{21} + \frac{\partial E}{\partial O_{12}} X_{22} + \frac{\partial E}{\partial O_{21}} X_{31} + \frac{\partial E}{\partial O_{22}} X_{32}$$

$$\frac{\partial E}{\partial F_{22}} = \frac{\partial E}{\partial O_{11}} X_{22} + \frac{\partial E}{\partial O_{12}} X_{23} + \frac{\partial E}{\partial O_{21}} X_{32} + \frac{\partial E}{\partial O_{22}} X_{33}$$

Backpropagation – Convolution Layer

- How to compute $\frac{\partial E}{\partial X_{11}}$?

Chain Rule

$$\frac{\partial E}{\partial X_{11}} = \left(\frac{\partial E}{\partial \boldsymbol{O}}\right)^T \frac{\partial \boldsymbol{O}}{\partial X_{11}}$$

$$\frac{\partial E}{\partial \boldsymbol{O}} = \nabla E(O_{11}, O_{12}, O_{21}, O_{22})$$

$$= \left(\frac{\partial E}{\partial O_{11}}, \frac{\partial E}{\partial O_{12}}, \frac{\partial E}{\partial O_{21}}, \frac{\partial E}{\partial O_{22}}\right)^T$$

$$\boldsymbol{O} = f(\boldsymbol{F}, \boldsymbol{X}) = (O_{11}, O_{12}, O_{21}, O_{22})^T \triangleq \left(f_1(\boldsymbol{F}, \boldsymbol{X}), f_2(\boldsymbol{F}, \boldsymbol{X}), f_3(\boldsymbol{F}, \boldsymbol{X}), f_4(\boldsymbol{F}, \boldsymbol{X})\right)^T$$

$$\frac{\partial \boldsymbol{O}}{\partial X_{11}} = \left(\frac{\partial f_1}{\partial X_{11}}, \frac{\partial f_2}{\partial X_{11}}, \frac{\partial f_3}{\partial X_{11}}, \frac{\partial f_4}{\partial X_{11}}\right)^T$$

Backpropagation – Convolution Layer

$$\boldsymbol{O} = f(\boldsymbol{F};\boldsymbol{X}) \quad f_1(\boldsymbol{X}) = O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$f_2(\boldsymbol{X}) = O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$f_3(\boldsymbol{X}) = O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$f_4(\boldsymbol{X}) = O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$

$$\frac{\partial E}{\partial X_{11}} = \frac{\partial E}{\partial O_{11}}F_{11} + \frac{\partial E}{\partial O_{12}}0 + \frac{\partial E}{\partial O_{21}}0 + \frac{\partial E}{\partial O_{22}}0$$

$$\frac{\partial E}{\partial X_{12}} = \frac{\partial E}{\partial O_{11}}F_{12} + \frac{\partial E}{\partial O_{12}}F_{11} + \frac{\partial E}{\partial O_{21}}0 + \frac{\partial E}{\partial O_{22}}0$$

$$\frac{\partial E}{\partial X_{13}} = \frac{\partial E}{\partial O_{11}}0 + \frac{\partial E}{\partial O_{12}}F_{12} + \frac{\partial E}{\partial O_{21}}0 + \frac{\partial E}{\partial O_{22}}0$$

$$\frac{\partial E}{\partial X_{21}} = \frac{\partial E}{\partial O_{11}}F_{21} + \frac{\partial E}{\partial O_{12}}0 + \frac{\partial E}{\partial O_{21}}F_{11} + \frac{\partial E}{\partial O_{22}}0$$

$$\frac{\partial E}{\partial X_{22}} = \frac{\partial E}{\partial O_{11}}F_{22} + \frac{\partial E}{\partial O_{12}}F_{21} + \frac{\partial E}{\partial O_{21}}F_{12} + \frac{\partial E}{\partial O_{22}}F_{11}$$

$$\frac{\partial E}{\partial X_{23}} = \frac{\partial E}{\partial O_{11}}0 + \frac{\partial E}{\partial O_{12}}F_{22} + \frac{\partial E}{\partial O_{21}}0 + \frac{\partial E}{\partial O_{22}}F_{11}$$

$$\frac{\partial E}{\partial X_{31}} = \frac{\partial E}{\partial O_{11}}0 + \frac{\partial E}{\partial O_{12}}0 + \frac{\partial E}{\partial O_{21}}F_{21} + \frac{\partial E}{\partial O_{22}}0$$

$$\frac{\partial E}{\partial X_{32}} = \frac{\partial E}{\partial O_{11}}0 + \frac{\partial E}{\partial O_{12}}0 + \frac{\partial E}{\partial O_{21}}F_{22} + \frac{\partial E}{\partial O_{22}}F_{21}$$

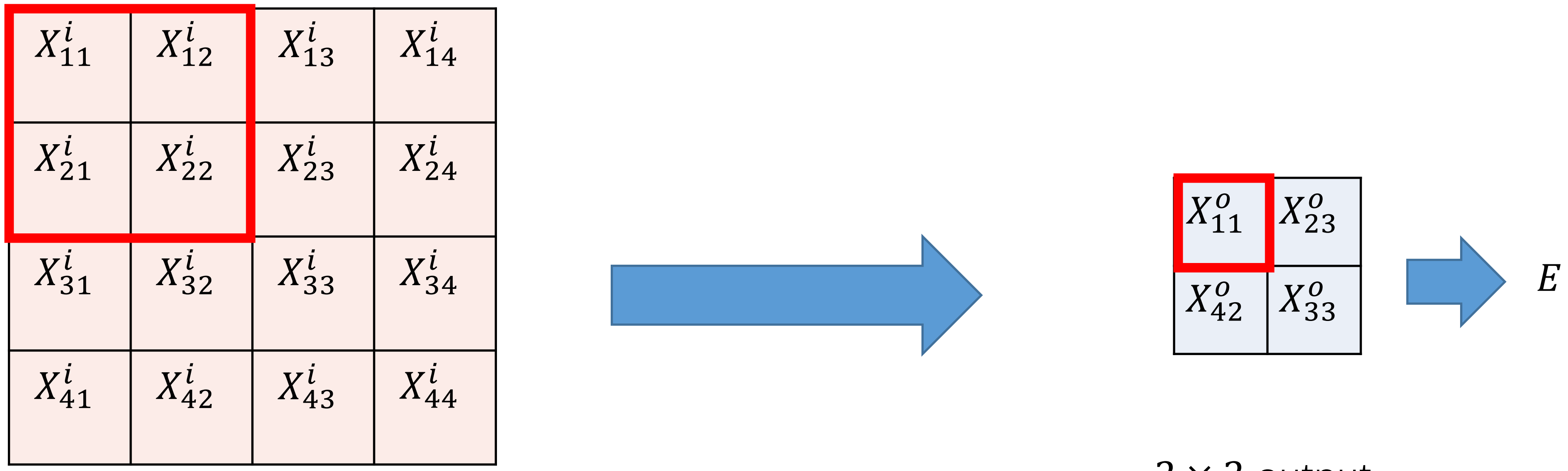
$$\frac{\partial E}{\partial X_{33}} = \frac{\partial E}{\partial O_{11}}0 + \frac{\partial E}{\partial O_{12}}0 + \frac{\partial E}{\partial O_{21}}0 + \frac{\partial E}{\partial O_{22}}F_{22}$$

Backpropagation – Convolution Layer

$$
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial E}{\partial O_{11}} & \frac{\partial E}{\partial O_{12}} & 0 \\ 0 & \frac{\partial E}{\partial O_{21}} & \frac{\partial E}{\partial O_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
+
\begin{bmatrix} F_{22} & F_{21} \\ F_{12} & F_{11} \end{bmatrix}
\Rightarrow
\begin{bmatrix} \frac{\partial E}{\partial X_{11}} & \frac{\partial E}{\partial X_{12}} & \frac{\partial E}{\partial X_{13}} \\ \frac{\partial E}{\partial X_{21}} & \frac{\partial E}{\partial X_{22}} & \frac{\partial E}{\partial X_{23}} \\ \frac{\partial E}{\partial X_{31}} & \frac{\partial E}{\partial X_{32}} & \frac{\partial E}{\partial X_{33}} \end{bmatrix}
$$

Backpropagation – Pooling Layer

- Max Pooling



4×4 Input

2×2 output

- Only need to compute the gradients of the inputs

$$\frac{\partial E}{\partial X^i_{11}} = \frac{\partial E}{\partial X^o_{11}} \qquad \frac{\partial E}{\partial X^i_{12}} = 0 \qquad \frac{\partial E}{\partial X^i_{21}} = 0 \qquad \frac{\partial E}{\partial X^i_{22}} = 0$$

$$\frac{\partial E}{\partial X^i_{13}} = 0 \qquad \frac{\partial E}{\partial X^i_{14}} = 0 \qquad \frac{\partial E}{\partial X^i_{23}} = \frac{\partial E}{\partial X^o_{23}} \qquad \frac{\partial E}{\partial X^i_{24}} = 0$$

$$\vdots$$

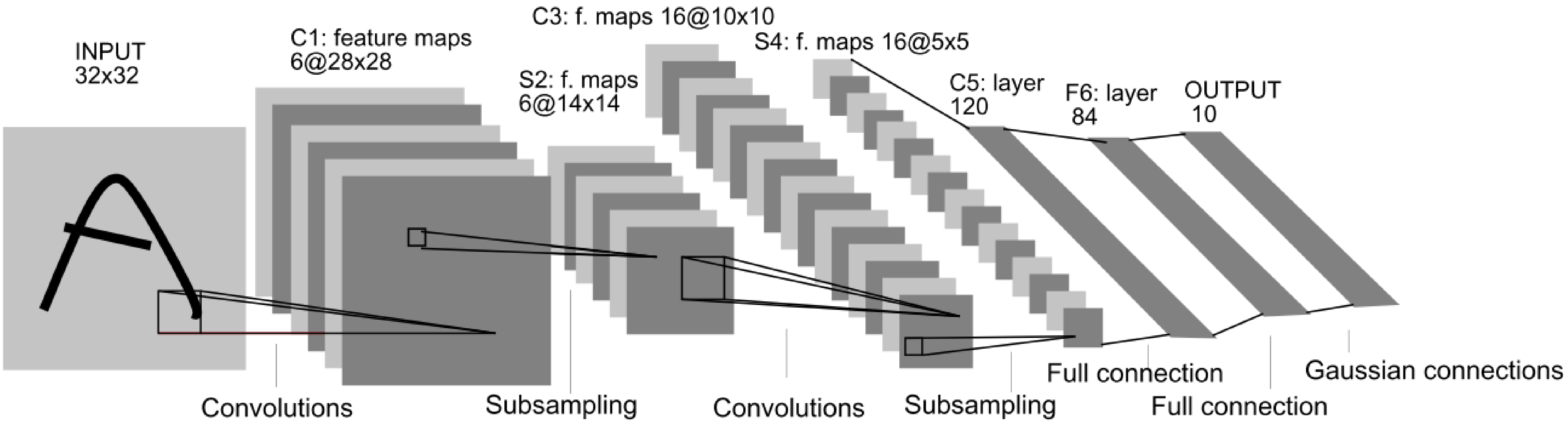
We assign the error to where it comes from

Examples of CNN Architecture

LeNet (1998)

- Gradient-based learning applied to document recognition

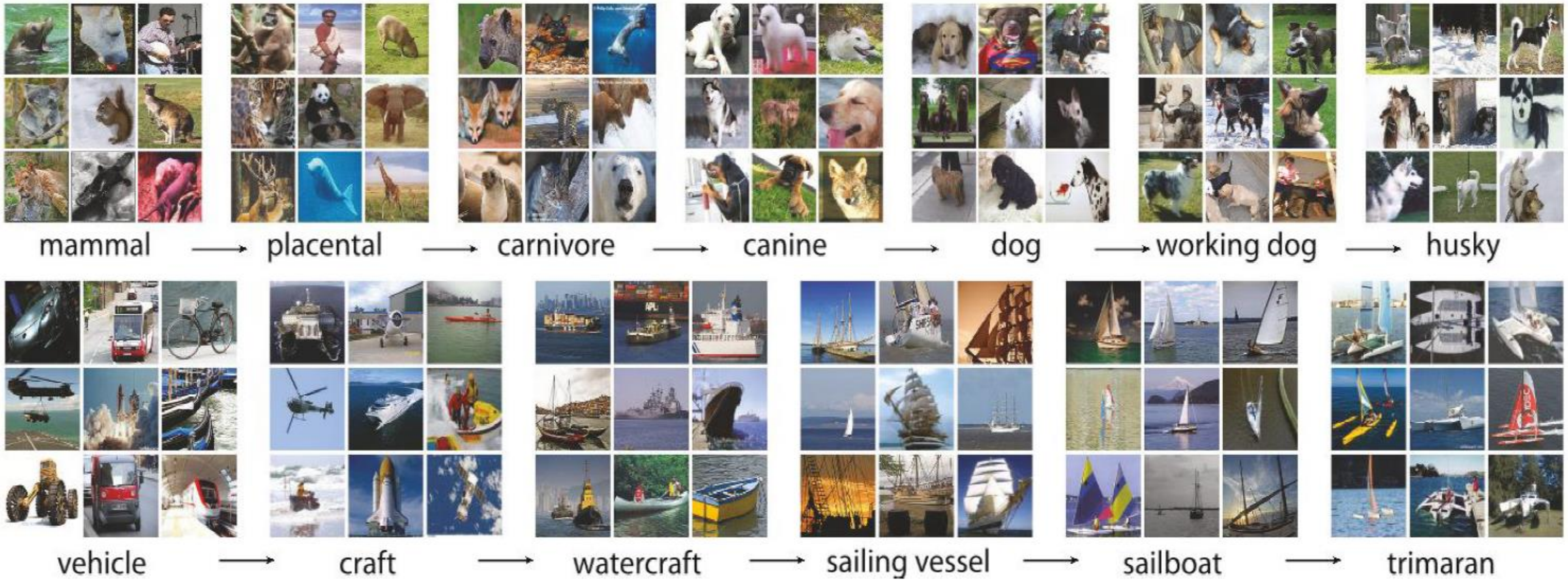
[Yann LeCun, Leon Bottou, Yoshua Bengio, and Patrick Haffner 1998]



LeNet-5 **5 layers**

ImageNet (Benchmark dataset)

- ImageNet
  - About 1.5×10^7 images, 2.2×10^4 categories
  - An image database organized according to the WordNet hierarchy



J Deng, W Dong, R Socher, LJ Li, K Li, L Fei-Fei. ImageNet: A large-scale hierarchical image. *CVPR*, 2009.

ISVRC

Steel drum

Output:
Scale
T-shirt
<u>Steel drum</u>
Drumstick
Mud turtle

✔

Output:
Scale
T-shirt
Giant panda
Drumstick
Mud turtle

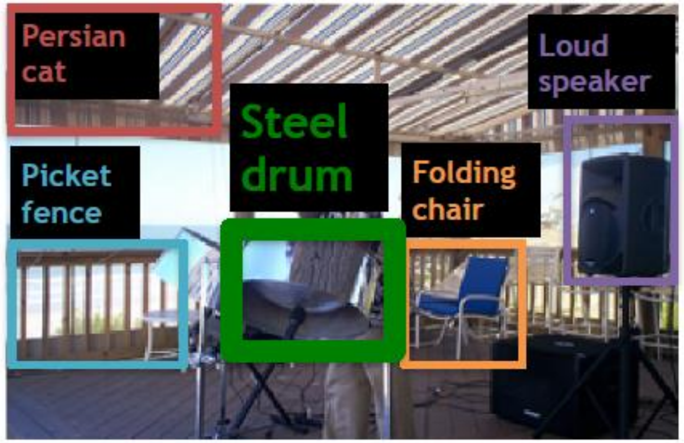
✗

Classification Task

Steel drum

✔

Output



Classification + Localization Task

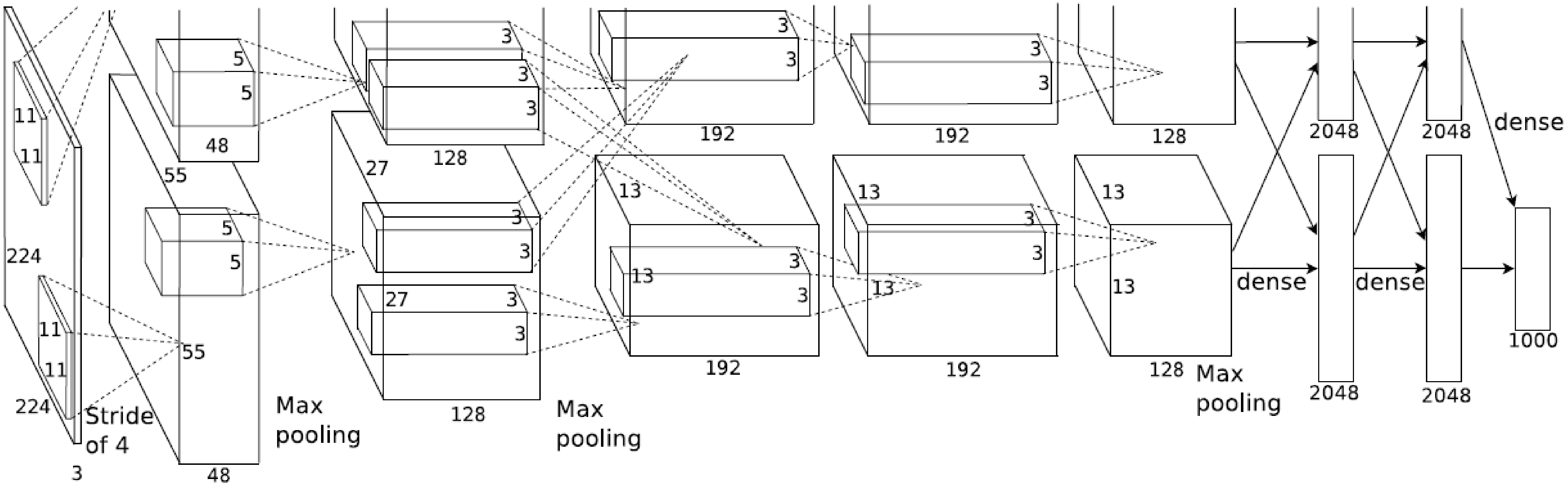
Deng et al. 2009, Russakovsky et al. 2015

Example credit: Fei-Fei Li and Jia Deng

AlexNet (2012)

- ImageNet Classification with Deep Convolutional Neural Networks

[Alex Krizhevsky, Ilya Sutskever, and Geoffrey E. Hinton 2012]



AlexNet **8 layers** **Top-5 Error rate: 16.4%**

VGGNet (2014)

- Very Deep Convolutional Networks for Large-Scale Image Recognition

[Karen Simonyan, and Andrew Zisserman 2014]

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

VGG-16 16 layers (trainable)

&

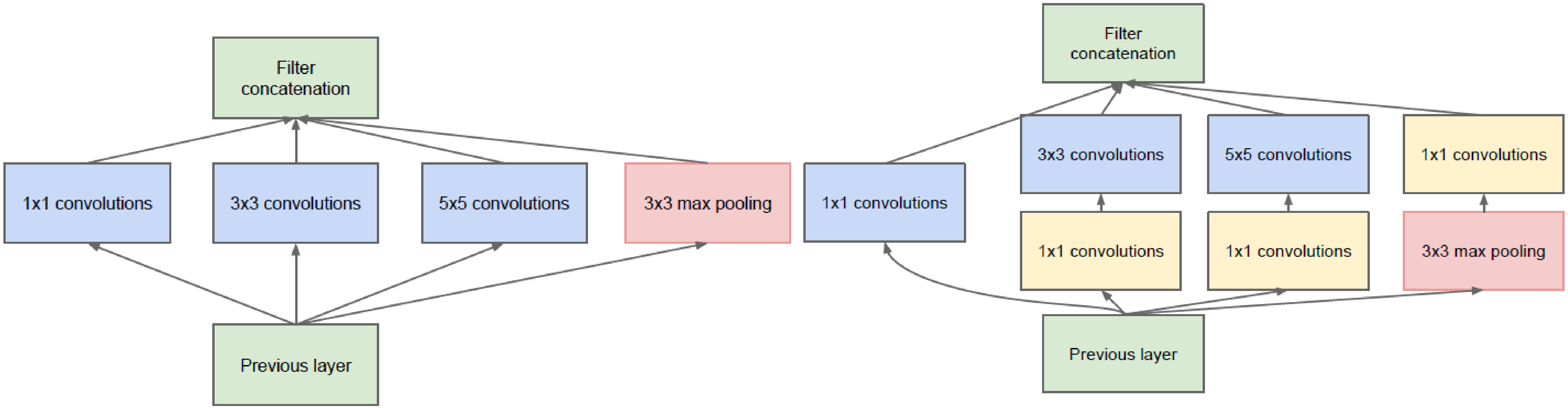
VGG-19 19 layers (trainable)

Top-5 Error rate: 7.3% (VGG-19)

GoogLeNet (2014)

- Going Deeper with Convolutions

[Karen Simonyan, Andrew Zisserman 2014]



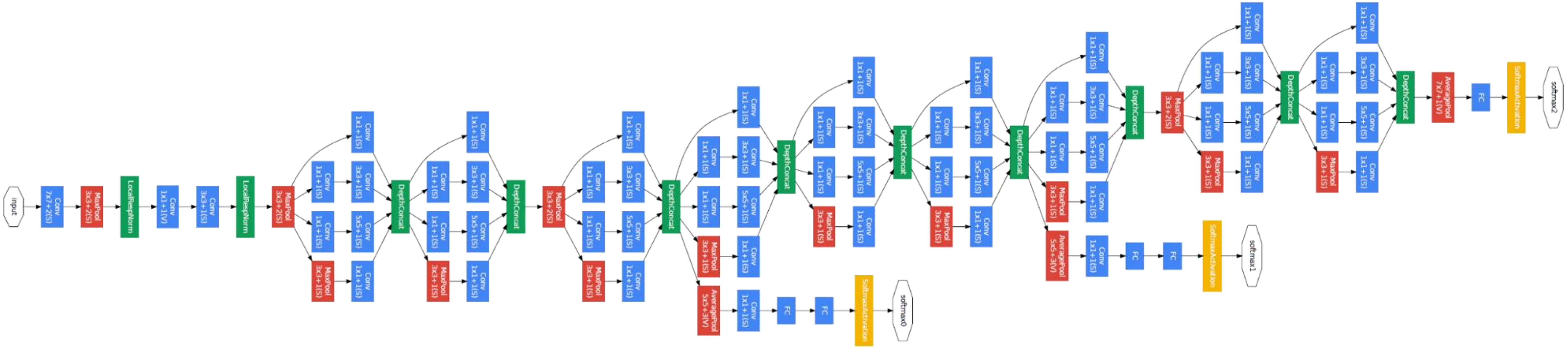
(a) Inception module, naïve version

(b) Inception module with dimensionality reduction

GoogLeNet (2014)

- Going Deeper with Convolutions

  [Karen Simonyan, Andrew Zisserman 2014]



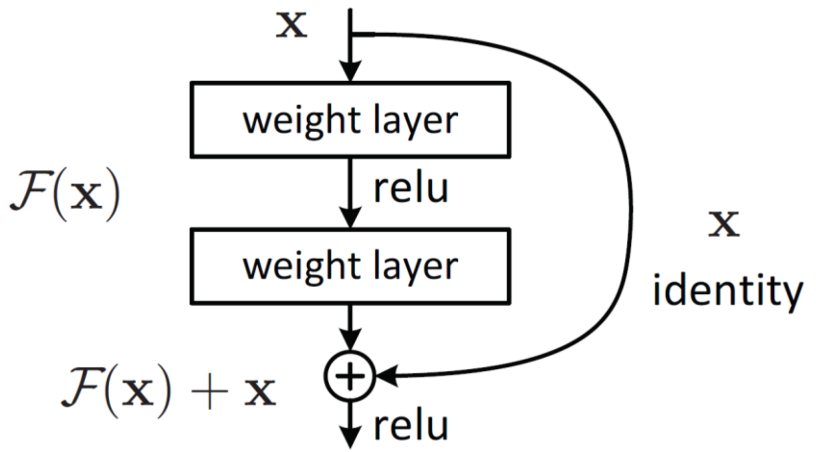
GoogLeNet

Only 3 layers??? **22 layers!!!** **Top-5 Error rate: 6.7%**

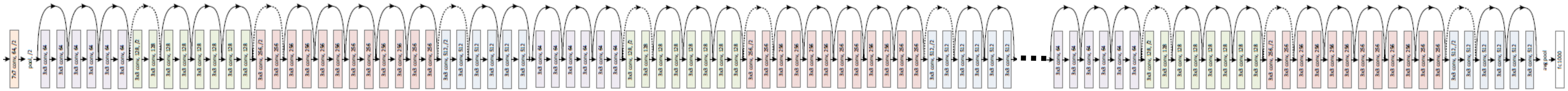
ResNet

- Deep Residual Learning for Image Recognition

[Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun 2015]



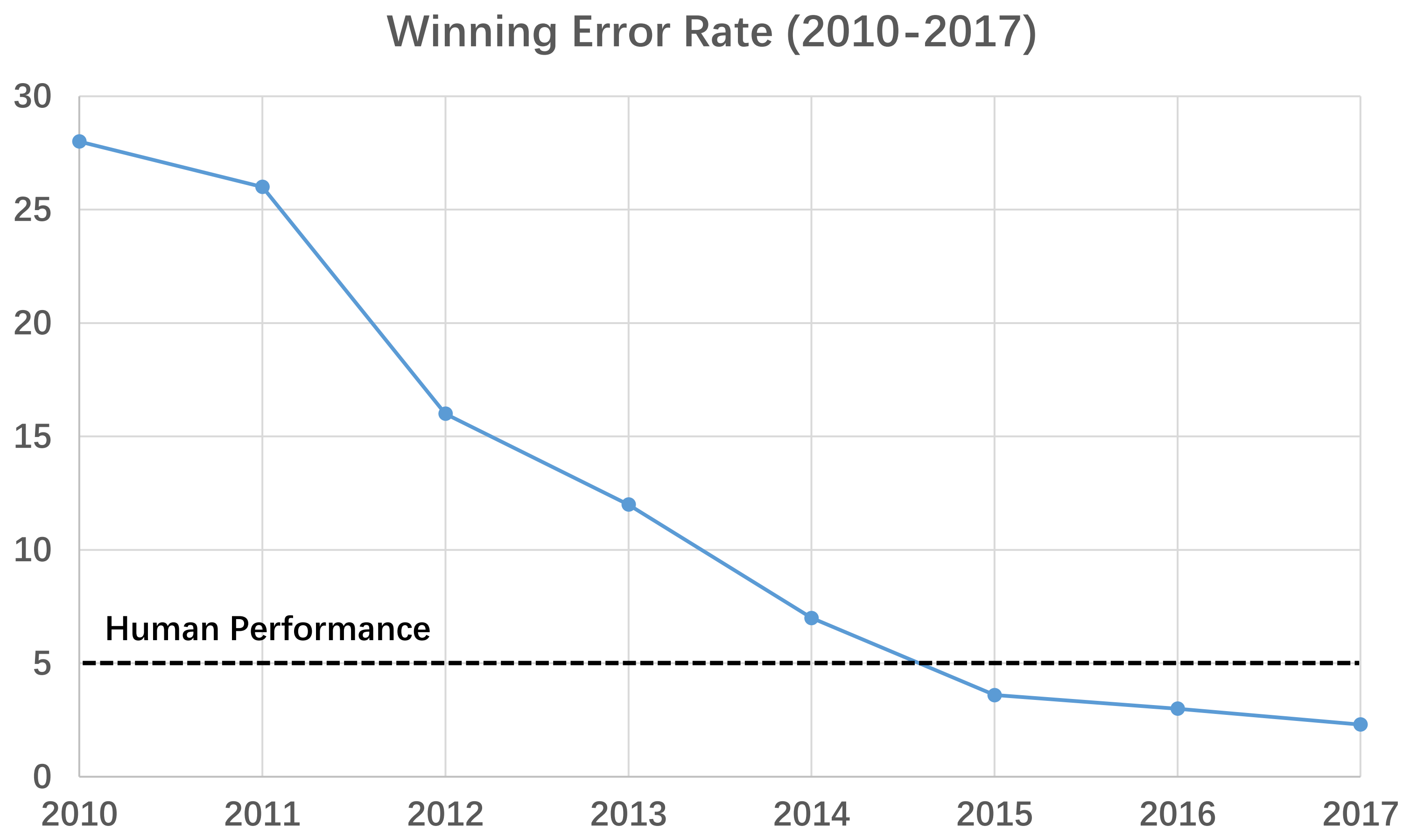
Residual learning: a building block



ResNet

Only 2 layers??? **Of course NOT!!!** **152 layers!!!** **Top-5 Error rate: 3.57%**

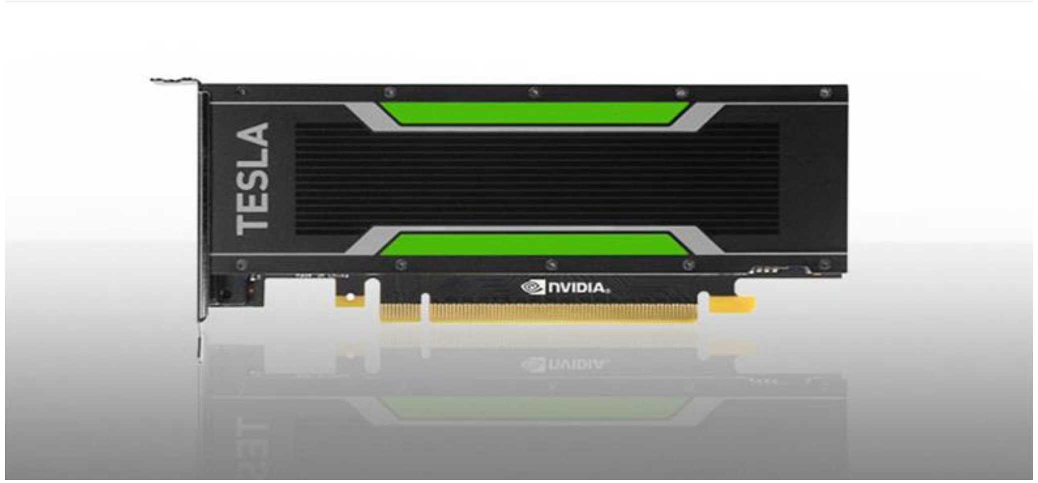
ResNet



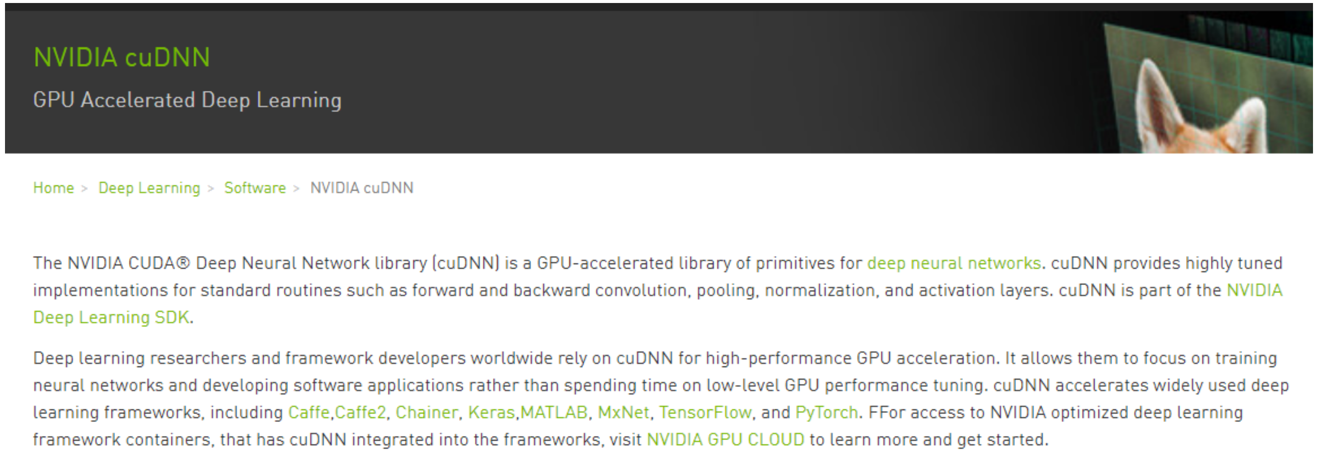
Winning Error Rate (2010-2017)
30
25
20
15
10
5
0
Human Performance
2010
2011
2012
2013
2014
2015
2016
2017

Hardware

- Trend
  - Deeper than deeper
  - Need a great number of computation resources



Hardware

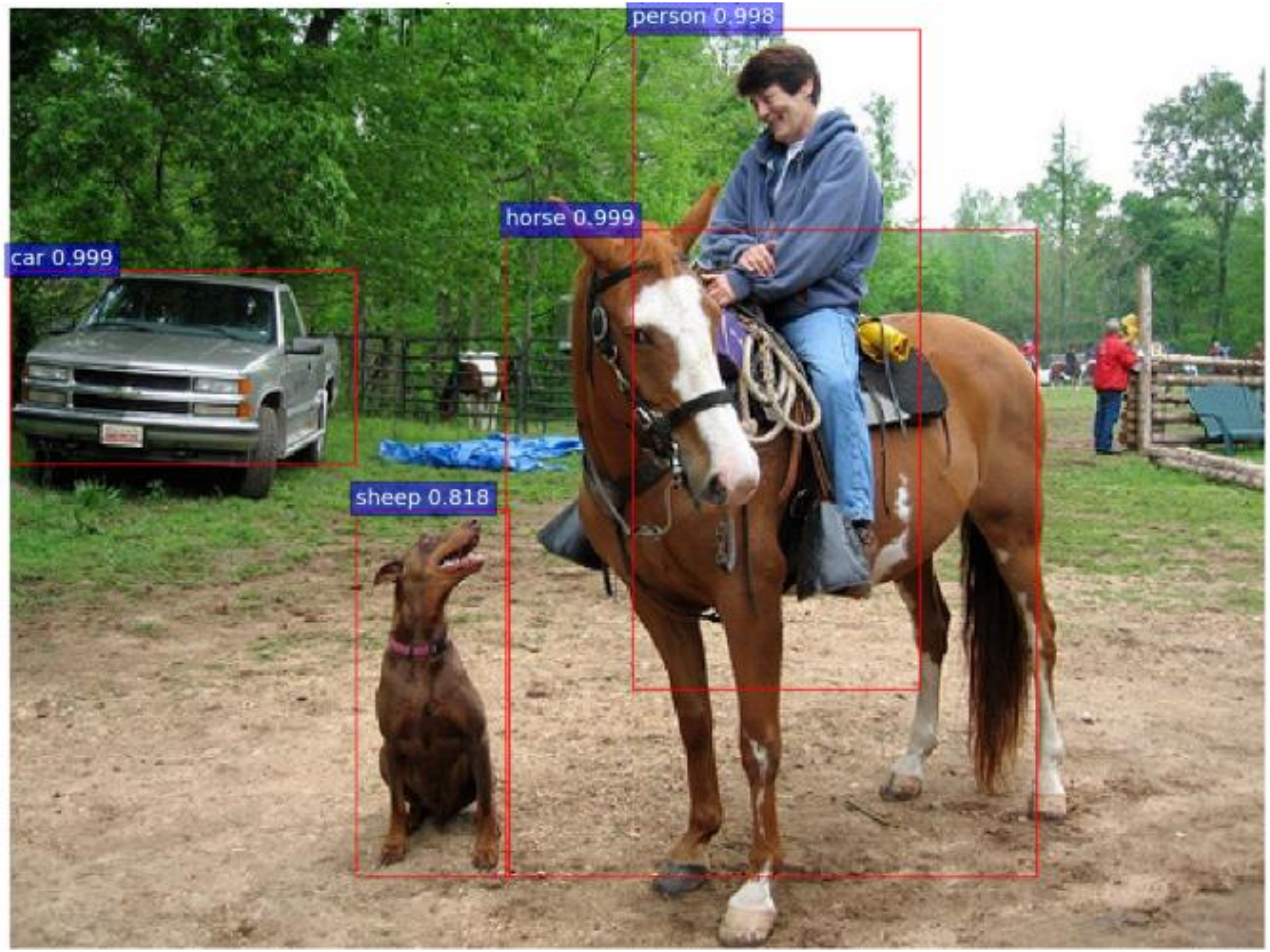


Software

Thanks to NVIDIA!

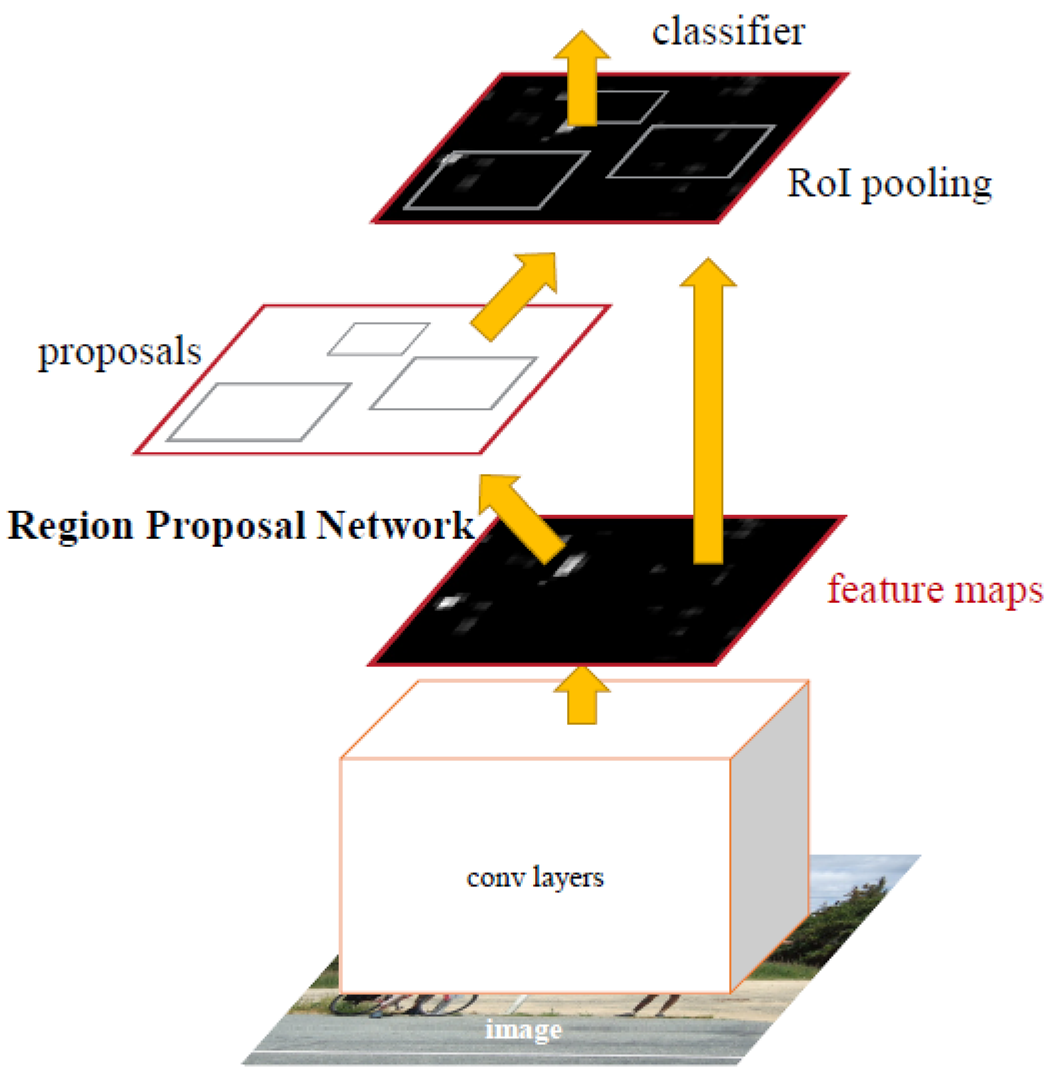
More Applications

Applications – Object Detection



Ren S, He K, Girshick R, et al. Faster R-CNN: Towards Real-Time Object Detection with Region Proposal Networks. NIPS 2015.

Applications – Object Detection

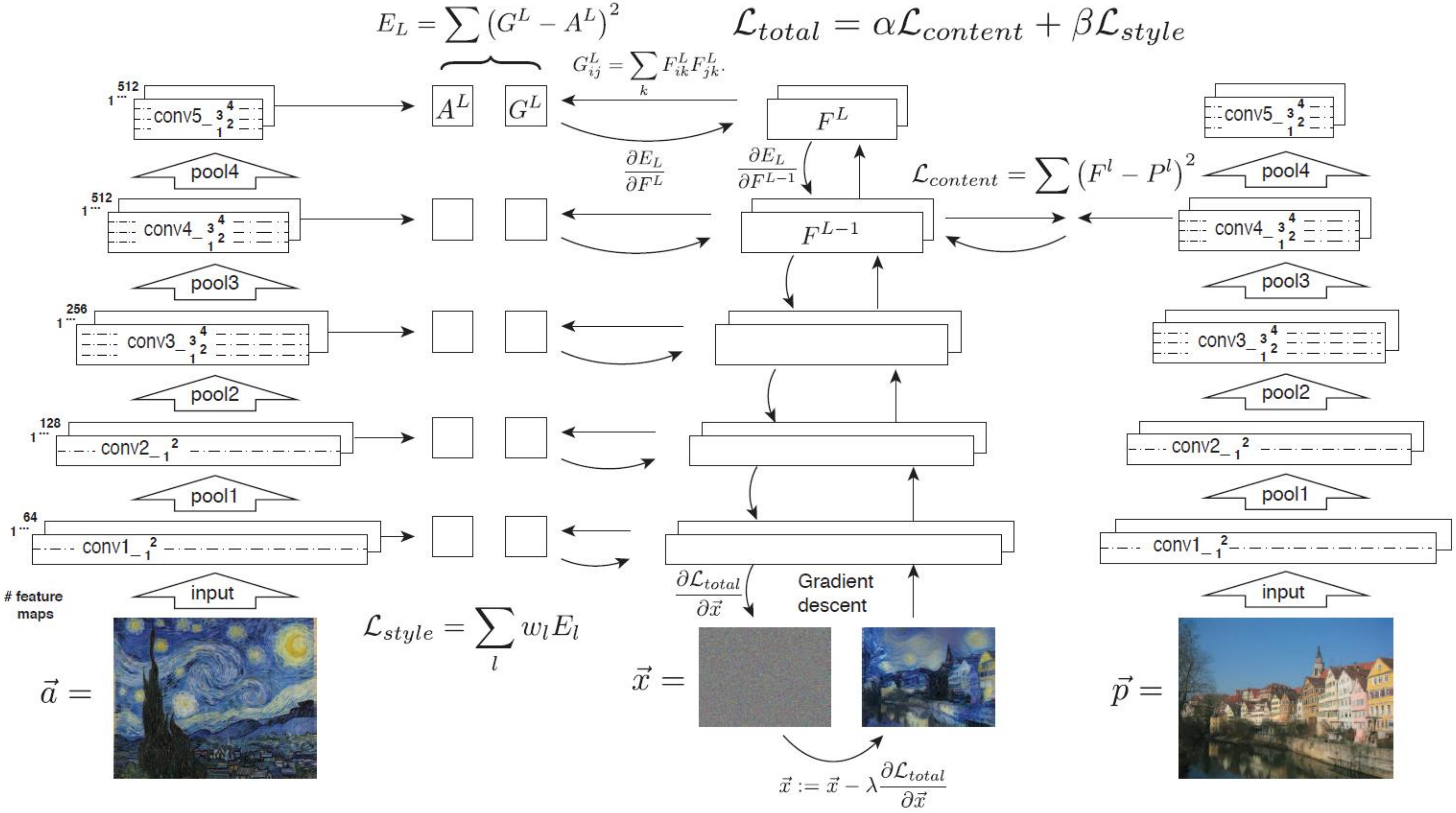


Ren S, He K, Girshick R, et al. Faster R-CNN: Towards Real-Time Object Detection with Region Proposal Networks. NIPS 2015.

Applications – Style Transfer

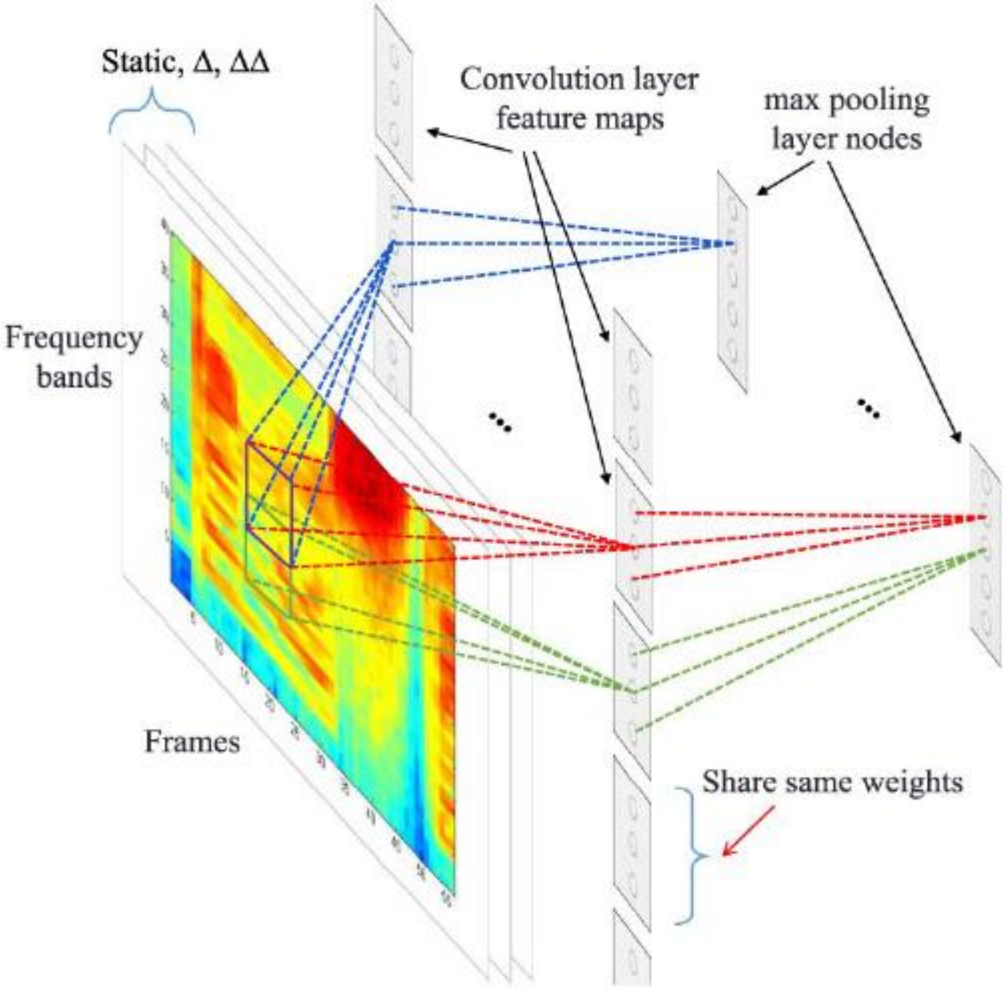
Gatys L A, Ecker A S, Bethge M. Image Style Transfer Using Convolutional Neural Networks. CVPR, 2016.

Applications – Style Transfer



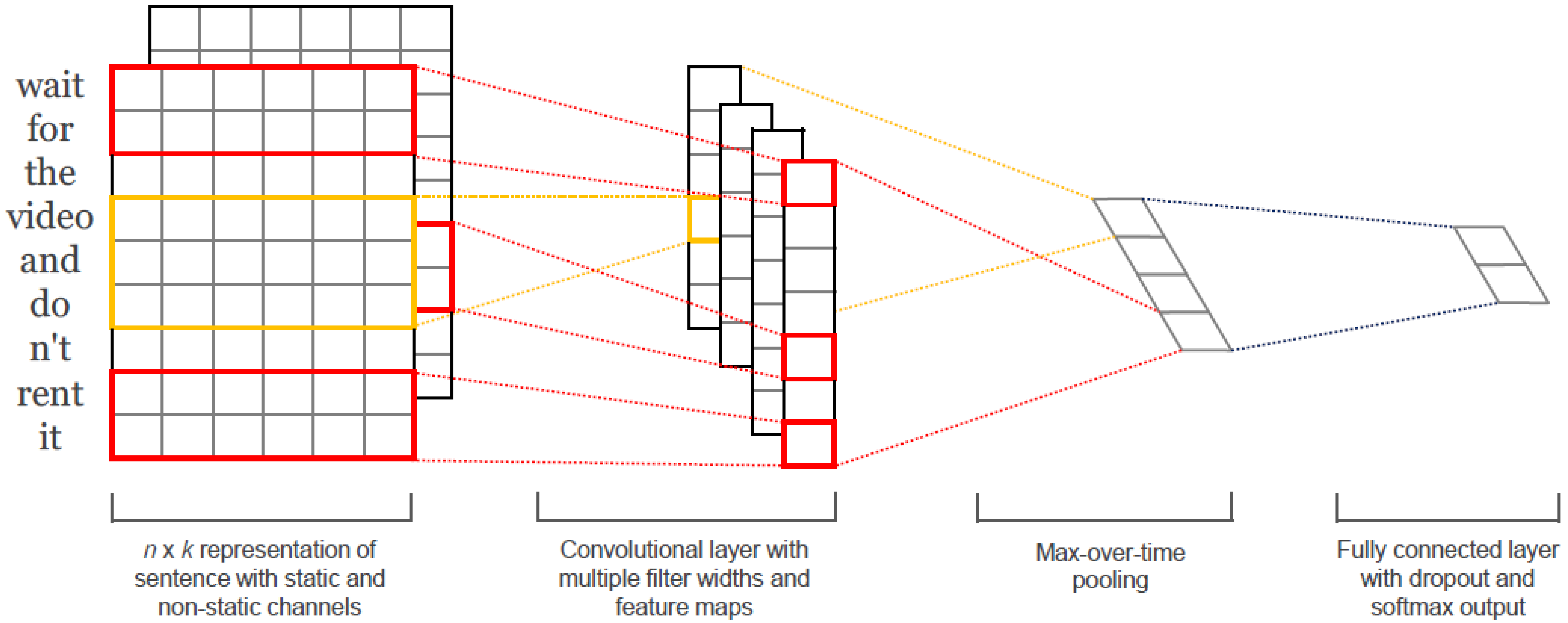
Gatys L A, Ecker A S, Bethge M. Image Style Transfer Using Convolutional Neural Networks. CVPR, 2016.

Applications – Speech Recognition



Abdel-Hamid O, Mohamed A R, Jiang H, et al. Convolutional Neural Networks for Speech Recognition[J]. IEEE/ACM Transactions on Audio Speech & Language Processing, 2014, 22(10):1533-1545.

Applications – Text Classification



Yoon Kim. Convolutional Neural Networks for Sentence Classification. EMNLP, 2014.

Materials

- Paper: Gradient-based Learning Applied to Document Recognition

  Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner, 1998.

- Paper: Deep Learning

  Y. Lecun, Y. Bengio, and G. Hinton, *Nature*, 2015.

- Course: Convolutional Neural Networks for Visual Recognition

  http://cs231n.stanford.edu/

- Tool: CNN Visualization

  https://blog.keras.io/how-convolutional-neural-networks-see-the-world.html

Questions