# Introduction to Machine Learning

Lecture 09: Decision Tree

Nov 8, 2022

Jie Wang

Machine Intelligence Research and Applications Lab

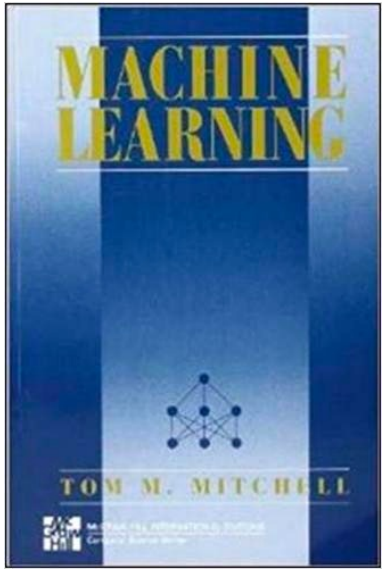
Department of Electronic Engineering and Information Science (EEIS)

http://staff.ustc.edu.cn/~jwangx/

jiewangx@ustc.edu.cn

# Contents

- **Example**
- **ID3**
- **Extensions of ID3**




Chapter 3

- **Example**

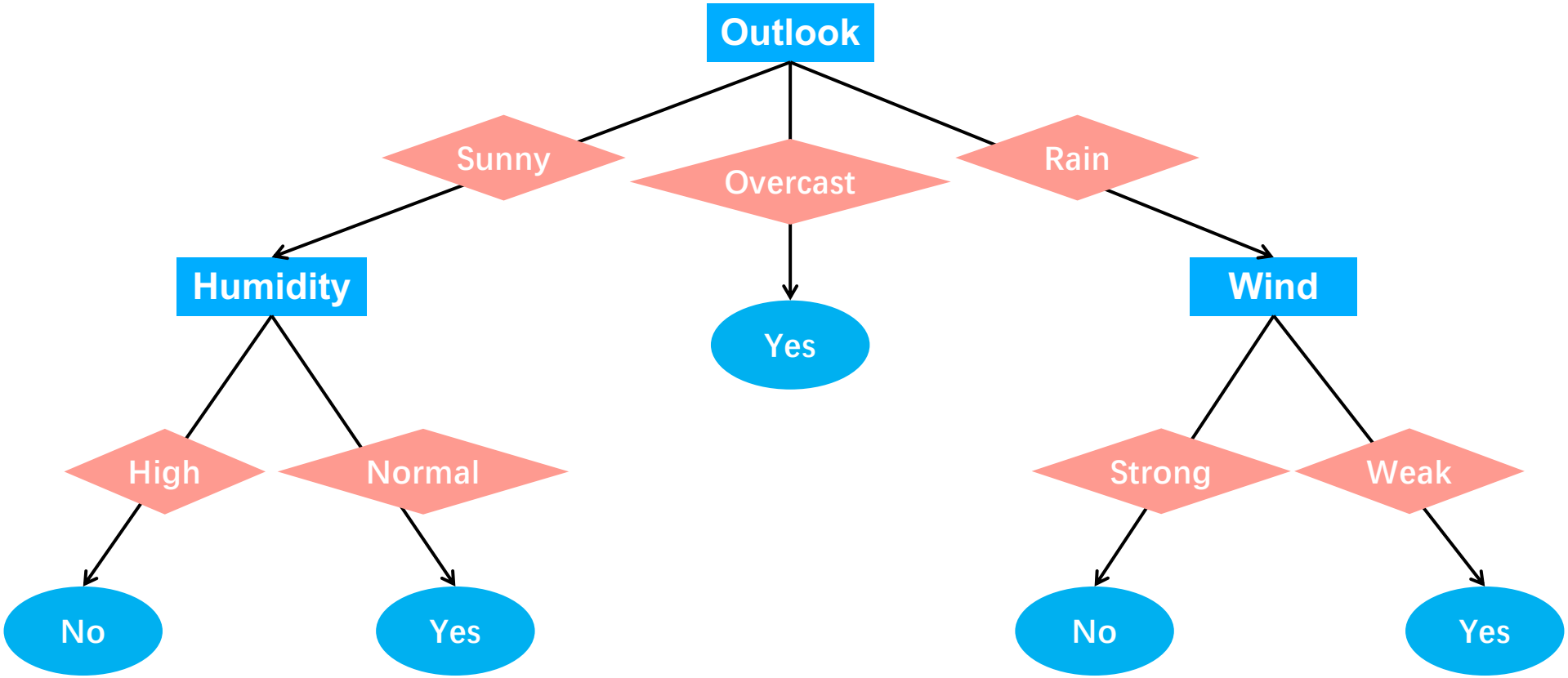
# Example

Features/attributes

labels

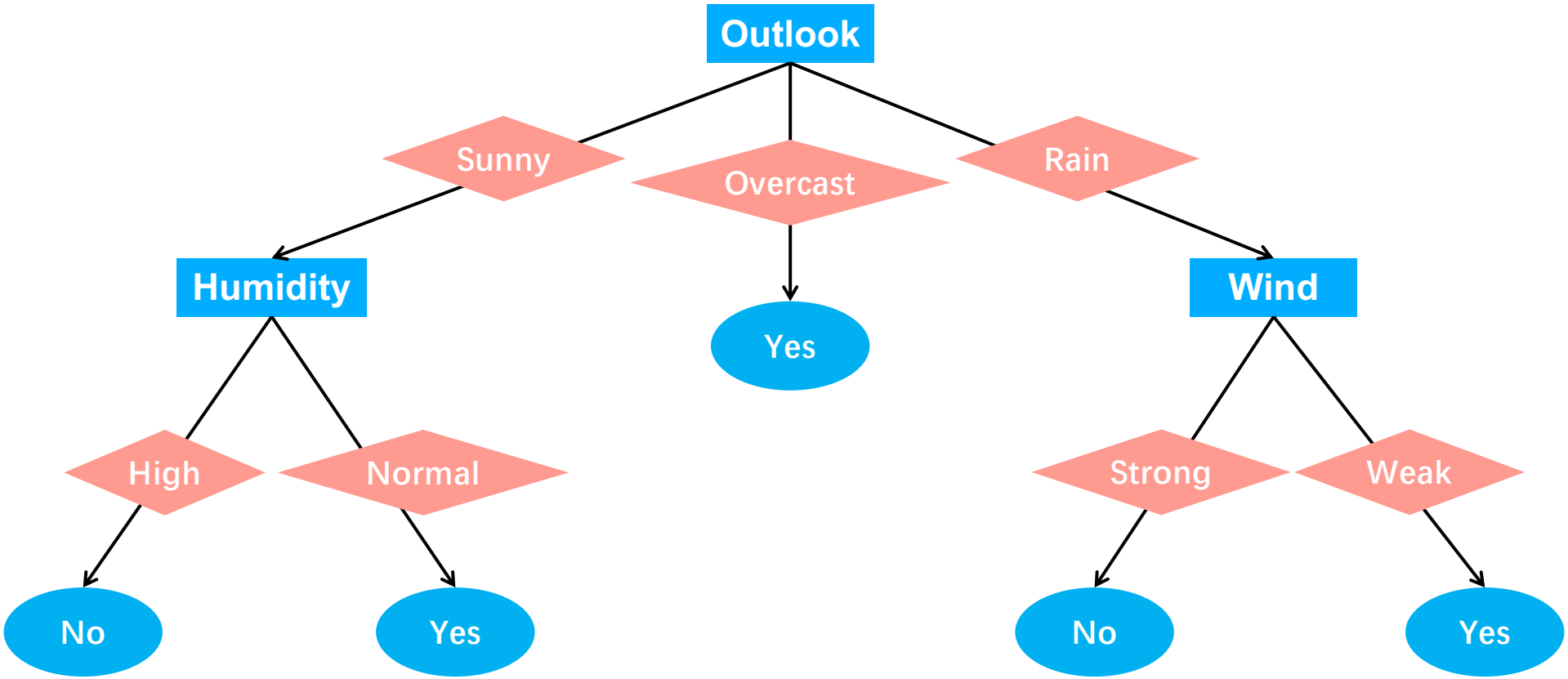
| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

# Example


Outlook
Sunny
Overcast
Rain
Humidity
Yes
Wind
High
Normal
Strong
Weak
No
Yes
No
Yes

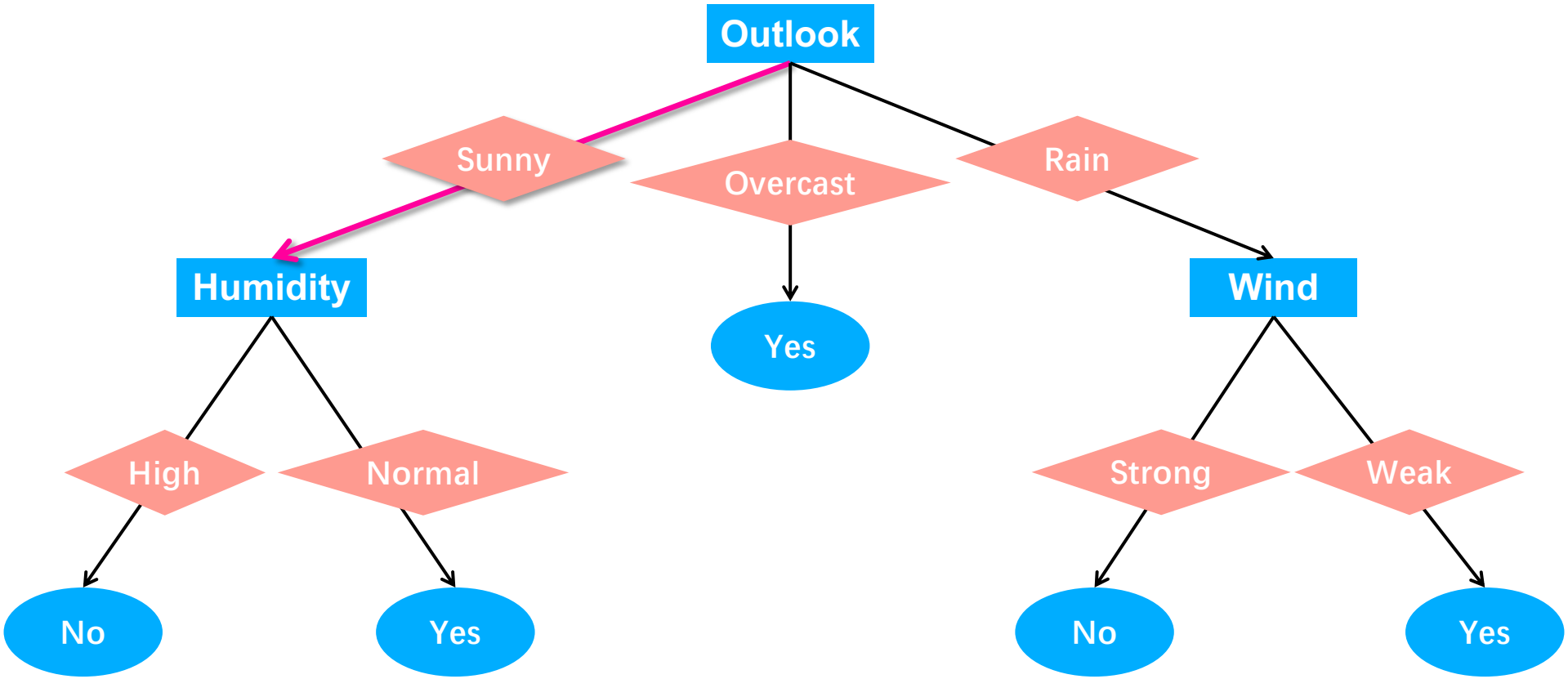
# Example

{Outlook=Sunny, Temperature=Hot, Humidity=High, Wind=Strong}



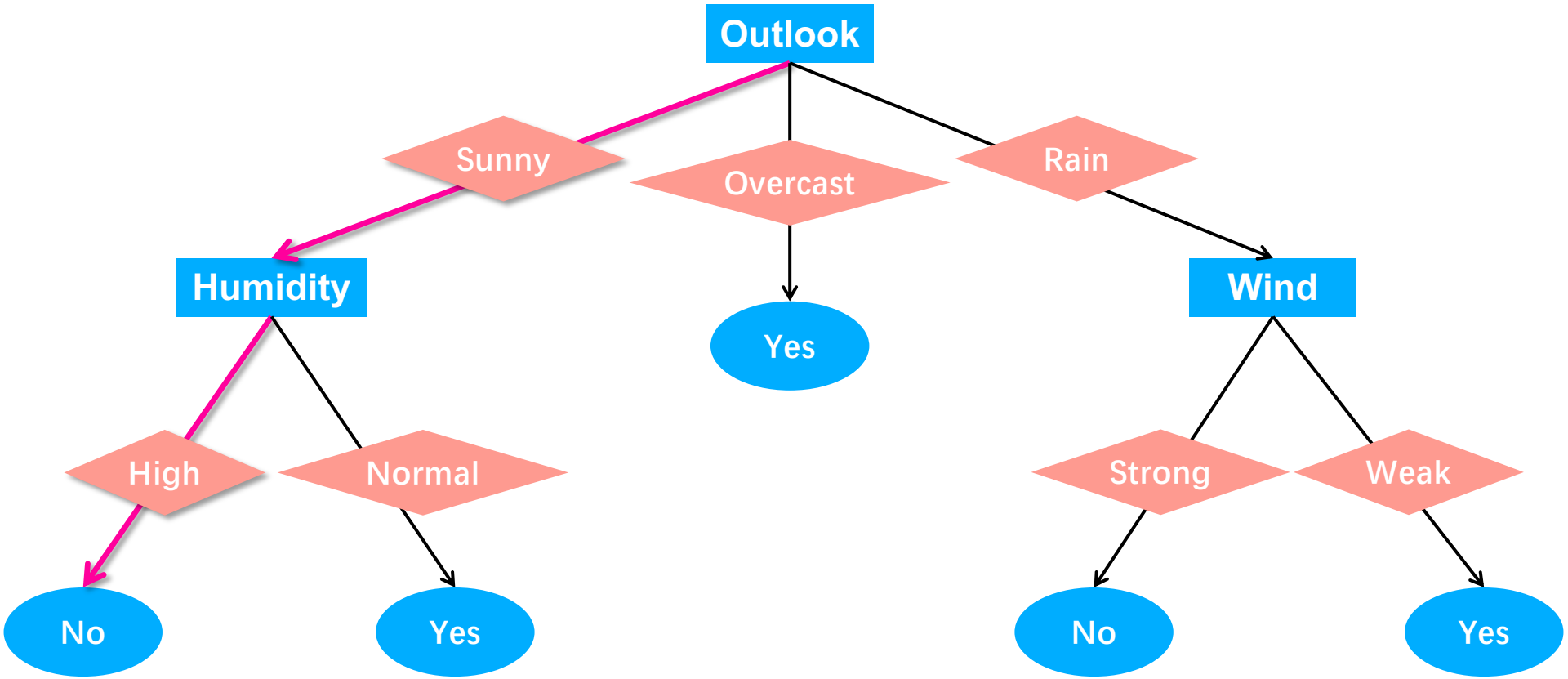
# Example

{Outlook=Sunny, Temperature=Hot, Humidity=High, Wind=Strong}



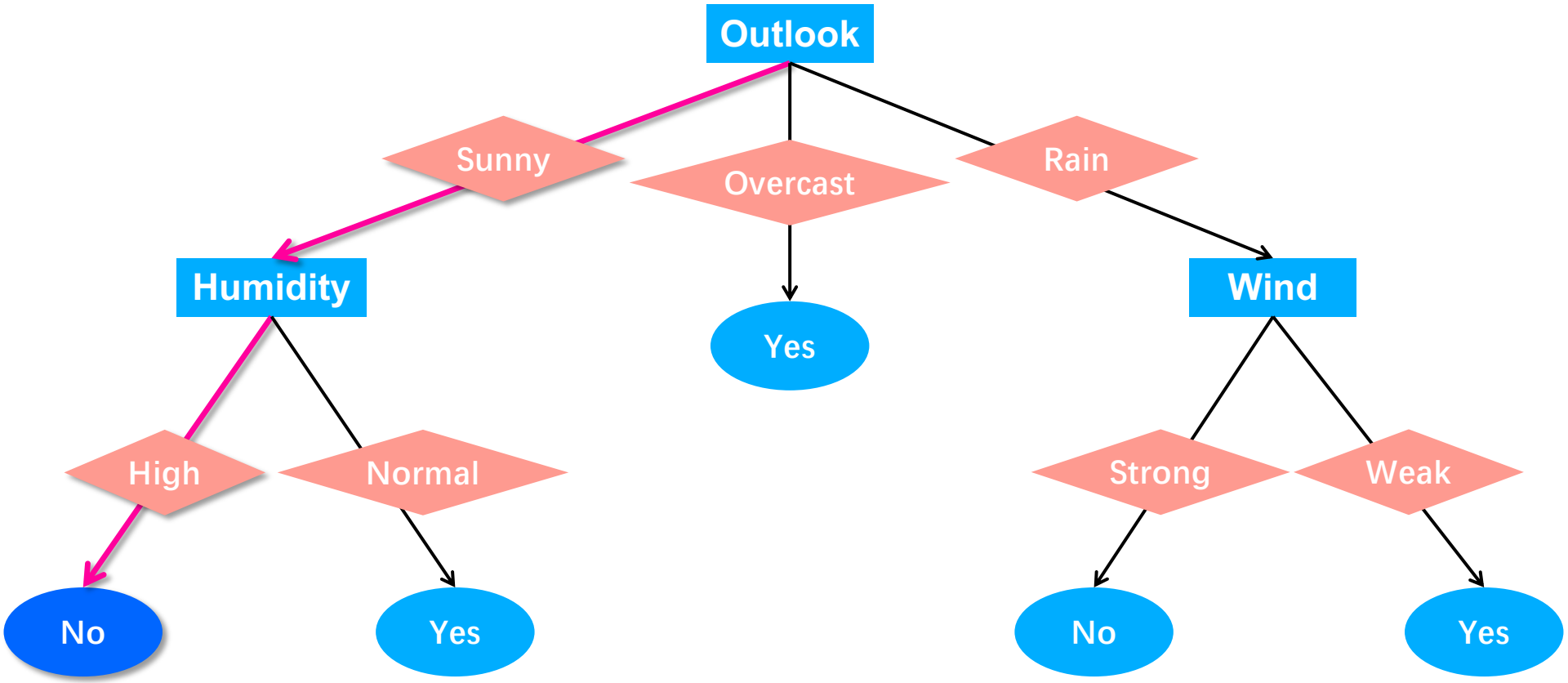
# Example

{Outlook=Sunny, Temperature=Hot, Humidity=High, Wind=Strong}



# Example

{Outlook=Sunny, Temperature=Hot, Humidity=High, Wind=Strong}



# Appropriate Problems

- Each attribute takes on a small number of disjoint possible values.
- The target function has discrete output values (classification).
- The training data may contain missing attribute values.
- ……

- **ID3**

# ID3

Which Attribute is the best classifier?

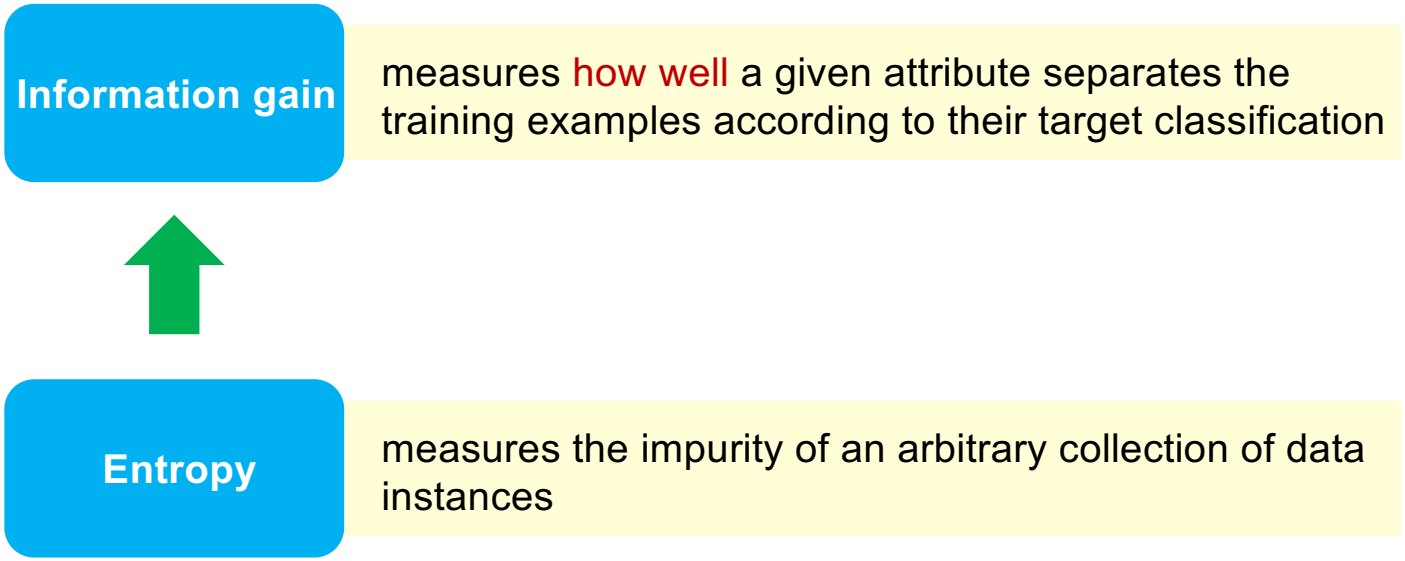
# ID3

Which Attribute is the best classifier?

**Information gain** measures how well a given attribute separates the training examples according to their target classification

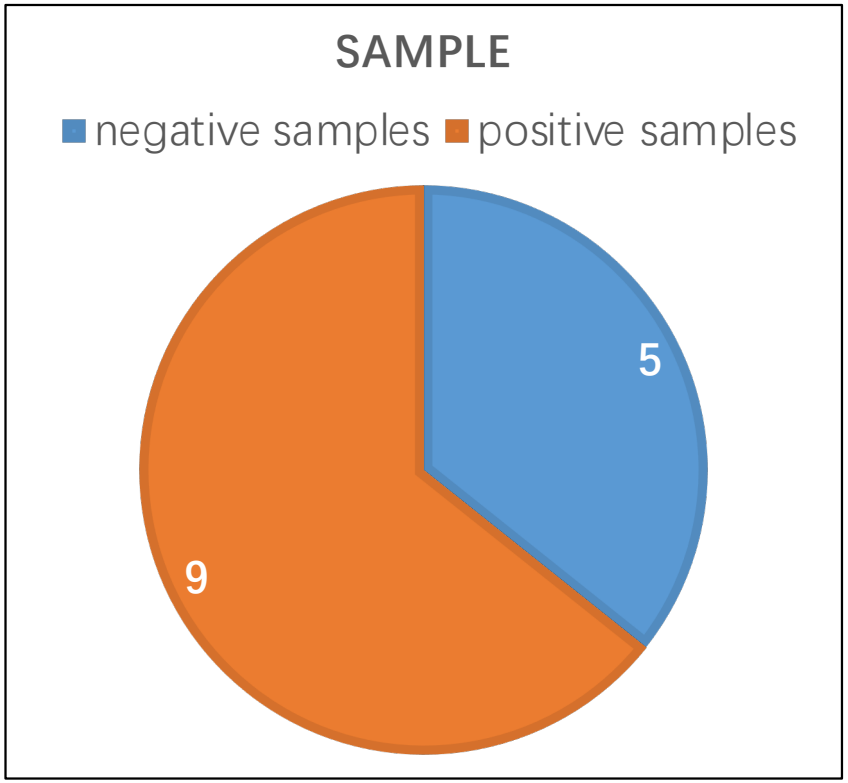
# ID3

- Which Attribute is the best classifier?



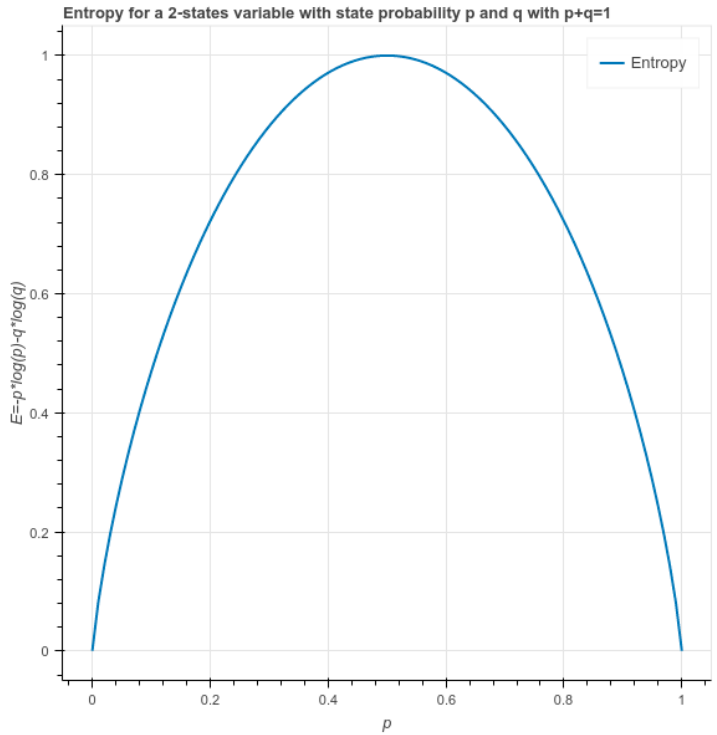
# Entropy

$$Entropy(S) := -p_+ \log_2 p_+ - p_- \log_2 p_-$$




$$\begin{aligned} & Entropy([9+, 5-]) \\ &= -(9/14)\log_2(9/14) - (5/14)\log_2(5/14) \\ &= 0.94 \end{aligned}$$

# Entropy




- The entropy is 0 if all members of S belong to the same class.
- The entropy is 1 when S contains an equal number of positive and negative examples.

https://bricaud.github.io/personal-blog/entropy-in-decision-trees/

# Information Gain

$$Gain(S, A) := Entropy(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

# Information Gain

$$Gain(S, A) := Entropy(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

$$Values(Wind) = \{Weak, Strong\}$$
$$S = [9+, 5-]$$
$$S_{Weak} \leftarrow [6+, 2-]$$
$$S_{Strong} \leftarrow [3+, 3-]$$

# Information Gain

$$Gain(S, A) := Entropy(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

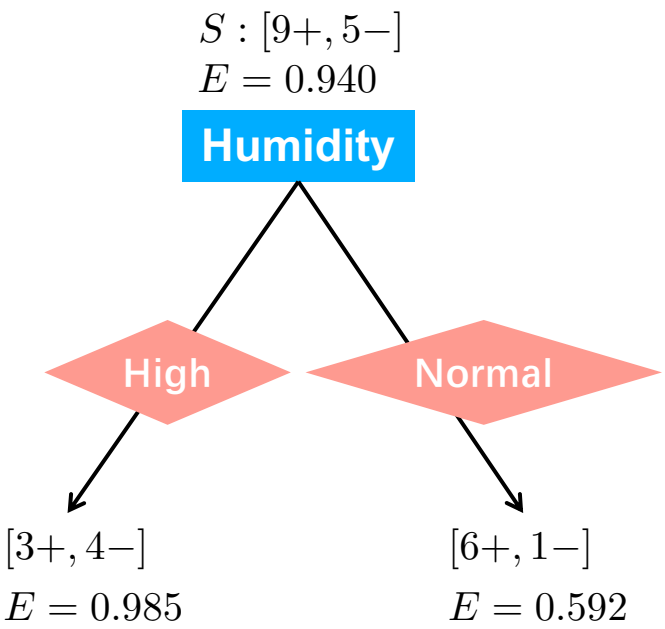
| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | Rain | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

$$Values(Wind) = \{Weak, Strong\}$$
$$S = [9+, 5-]$$
$$S_{Weak} \leftarrow [6+, 2-]$$
$$S_{Strong} \leftarrow [3+, 3-]$$

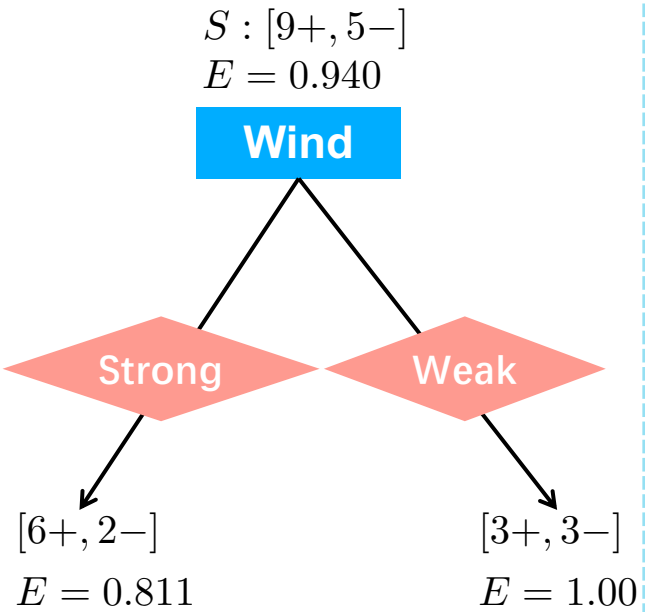
$$\begin{aligned} &Gain(S, Wind) \\ &= Entropy(S) - \sum_{v \in \{Weak, Strong\}} \frac{|S_v|}{|S|} Entropy(S_v) \\ &= Entropy(S) - (8/14) Entropy(S_{Weak}) \\ &\quad - (6/14) Entropy(S_{Strong}) \\ &= 0.940 - (8/14) 0.811 - (6/14) 1.00 \\ &= 0.048 \end{aligned}$$

# Information Gain

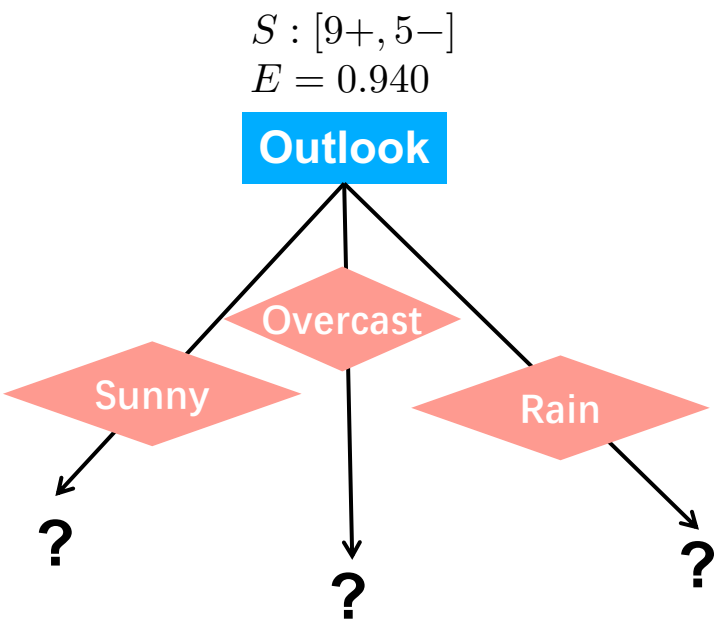
- Which Attribute is the best classifier?




$$Gain(S, Humidity)$$
$$=0.94 - (7/14)0.985 - (7/14)0.592$$
$$=0.151$$




$$Gain(S, Wind)$$
$$=0.94 - (8/14)0.811 - (6/14)1.0$$
$$=0.048$$




$$Gain(S, Outlook) = \ ?$$

# Information Gain

ID3($Examples$, $Target_attribute$, $Attributes$)

*Examples are the training examples. Target_attribute is the attribute whose value is to be predicted by the tree. Attributes is a list of other attributes that may be tested by the learned decision tree. Returns a decision tree that correctly classifies the given Examples.*

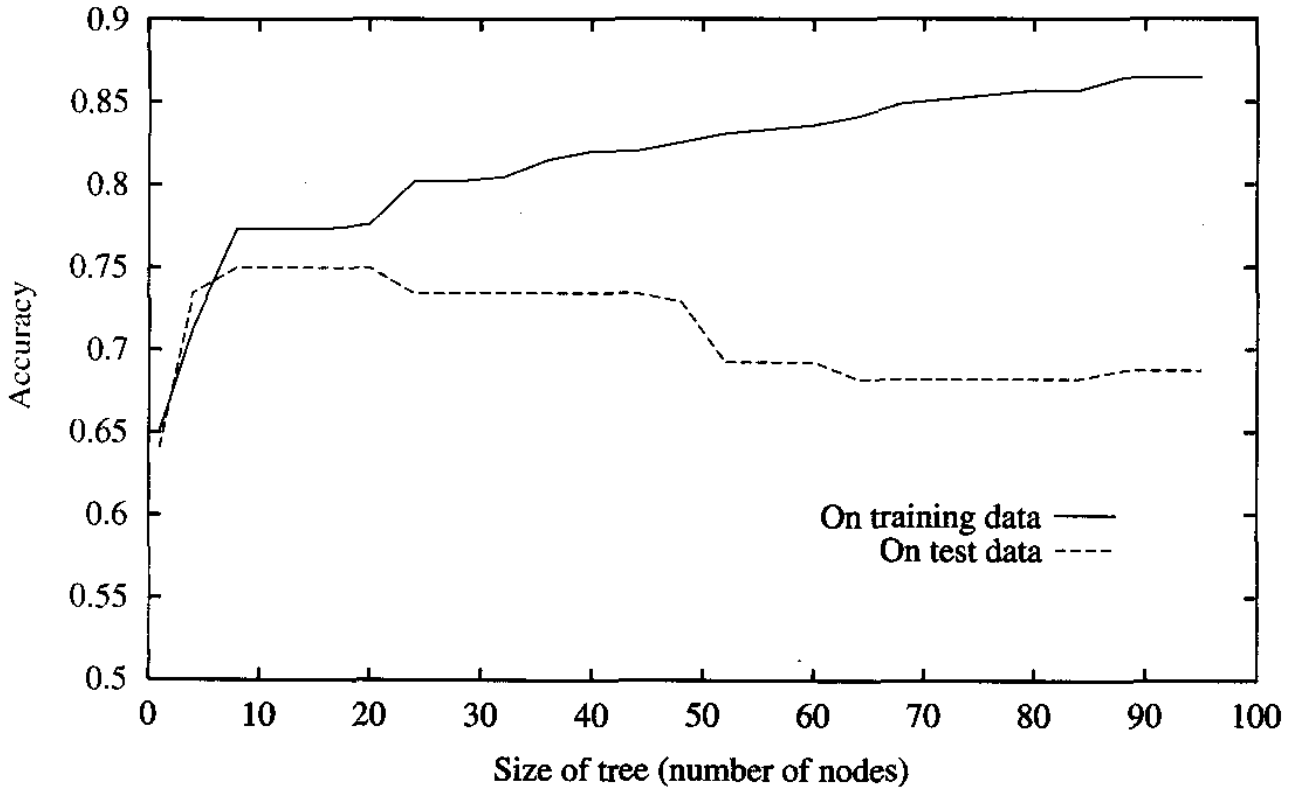
dealing with the corner cases

- Create a *Root* node for the tree
- If all *Examples* are positive, Return the single-node tree *Root*, with label = +
- If all *Examples* are negative, Return the single-node tree *Root*, with label = −
- If *Attributes* is empty, Return the single-node tree *Root*, with label = most common value of *Target_attribute* in *Examples*
- Otherwise Begin
  - $A \leftarrow$ the attribute from *Attributes* that best* classifies *Examples*
  - The decision attribute for *Root* $\leftarrow A$
  - For each possible value, $v_i$, of $A$,
    - Add a new tree branch below *Root*, corresponding to the test $A = v_i$
    - Let $Examples_{v_i}$ be the subset of *Examples* that have value $v_i$ for $A$
    - If $Examples_{v_i}$ is empty
      - Then below this new branch add a leaf node with label = most common value of *Target_attribute* in *Examples*
      - Else below this new branch add the subtree ID3($Examples_{v_i}$, $Target_attribute$, $Attributes - \{A\}$))
- End
- Return *Root*

For the tree constructed by ID3, we shall not see an attribute more than once along any paths.

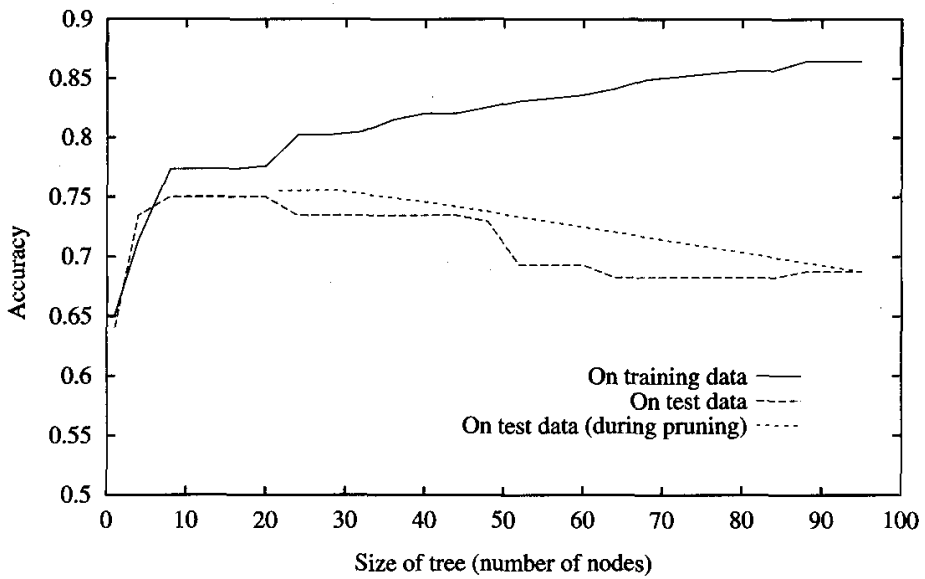
# Pruning

- Overfitting



# Pruning

- Post-pruning
  - Split the data into a training set and a validation set
  - Train the decision tree on the training set
  - **While** pruning improves the accuracy of the tree on the validation set
    - Scan the nodes one by one
    - **If** removing the nodes (and all its descendants) improves the accuracy of the tree on the validation set
      - Remove the node and all its descendants
    - **Endif**

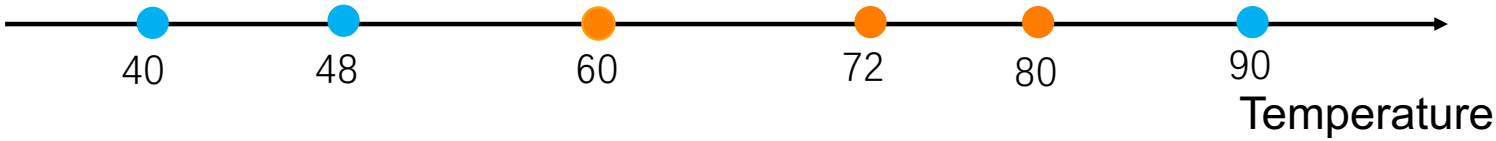


# Questions

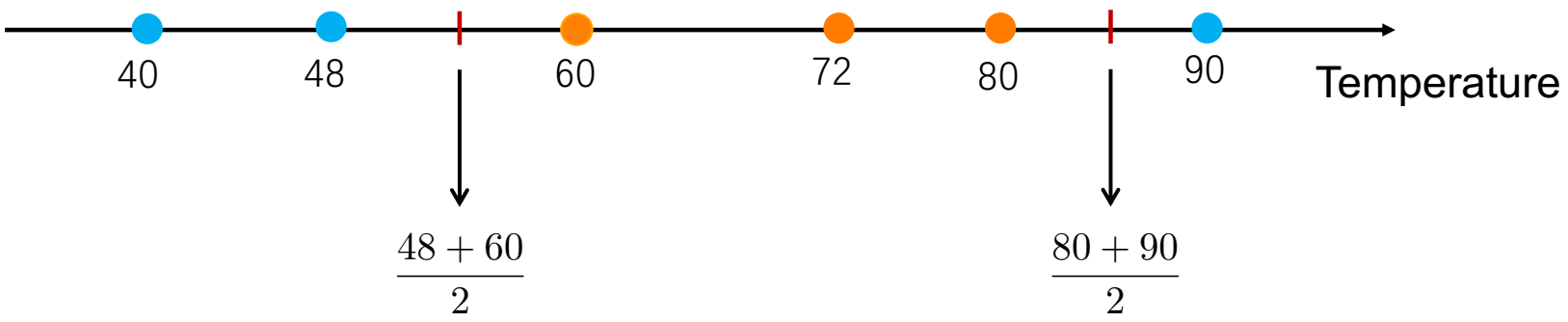
- Does there exist an attribute (may only in theory) that leads to the maximum information gain?
- Is the information gain always nonnegative?

- **Extensions of ID3**

# Continuous-Valued Attributes



# Continuous-Valued Attributes



# Missing Attribute Values

| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | ? | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

- Approach 1
  - Assign the common value to the missing attribute value

# Missing Attribute Values

| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | ? | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

- Approach 1
  - Assign the common value to the missing attribute value
- Approach 2
  - Weight the instance by the frequencies of the attribute values

# Missing Attribute Values

| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | ? | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

- Approach 1
  - Assign the common value to the missing attribute value
- Approach 2
  - Weight the instance by the frequencies of the attribute values

| D6 | ? | Cool | Normal | Strong | No |
|---|---|---|---|---|---|

# Missing Attribute Values

| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|---|---|---|---|---|---|
| D1 | Sunny | Hot | High | Weak | No |
| D2 | Sunny | Hot | High | Strong | No |
| D3 | Overcast | Hot | High | Weak | Yes |
| D4 | Rain | Mild | High | Weak | Yes |
| D5 | Rain | Cool | Normal | Weak | Yes |
| D6 | ? | Cool | Normal | Strong | No |
| D7 | Overcast | Cool | Normal | Strong | Yes |
| D8 | Sunny | Mild | High | Weak | No |
| D9 | Sunny | Cool | Normal | Weak | Yes |
| D10 | Rain | Mild | Normal | Weak | Yes |
| D11 | Sunny | Mild | Normal | Strong | Yes |
| D12 | Overcast | Mild | High | Strong | Yes |
| D13 | Overcast | Hot | Normal | Weak | Yes |
| D14 | Rain | Mild | High | Strong | No |

- Approach 1
  - Assign the common value to the missing attribute value
- Approach 2
  - Weight the instance by the frequencies of the attribute values

| | | | | | | |
|---|---|---|---|---|---|---|
| | D6 | ? | Cool | Normal | Strong | No |
| 5/13 | D6-1 | Sunny | Cool | Normal | Strong | No |
| 4/13 | D6-2 | Overcast | Cool | Normal | Strong | No |
| 4/13 | D6-3 | Rain | Cool | Normal | Strong | No |

# Resources

- http://www.r2d3.us/visual-intro-to-machine-learning-part-1/