

Problem

Please select a problem to answer. We will evaluate your math skills according to your answer.

Problem 1: Second-order sufficient optimality conditions

Suppose that $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is twice differentiable at \mathbf{x} . Show that \mathbf{x} is a strict local minimum if $\nabla f(\mathbf{x}) = 0$ and the Hessian matrix $\mathbf{H}(\mathbf{x})$ is positive definite.

Problem 2: Low-rank approximation

Please solve the problem as follows

$$\min_{X \in \mathbb{R}^{m \times n}} \{\|A - X\|_F : \mathbf{rank}(X) \leq K\}.$$

Problem 3: Random walk on \mathbb{Z}

Consider the random walk $X = \{X_n\}_{n \geq 0}$ on \mathbb{Z} that starts at $X_0 = 0$. The particle moves with probability p one unit to the right and with probability $q = 1 - p$ one unit to the left at each transition. Prove that the state 0 is recurrent (i.e., $\mathbb{P}\{X_n = 0 \text{ i.o.} | X_0 = 0\} = 1$) if and only if $p = q = 1/2$.